FINAL REPORT

IMA RESEARCH INCUBATOR GRANT

“Financial Statement Items as Interlinked Stochastic Processes”

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Introduction and Summary

As stated in the grant application, the long term objectives of this research are

- "to document the need to express financial statement items as interrelated stochastic processes, by highlighting the significant uncertainties associated with them…" and
- to "devise methodologies for estimating measurement error …, presenting these estimates, [and] using subsequent observations to update [them]".

When I first started thinking about this line of research, my original goal was to build a Monte Carlo simulation model of forward looking financial statements. Hence the title of my project.

In the course of my literature review, however, I discovered the dichotomy made by Harris *et al*, 2012 between "uncertainty around current value" and "risk", with the former being uncertainties surrounding the values on the balance sheet as given, and the latter being the forward looking financial statement items of my original interest. I decided that uncertainties around current values needed to be characterized first. Not only must these uncertainties be the starting point of any forward looking model, but the problem of how to effectively communicating them needs to be solved before the harder problem of communicating the uncertainties of forward looking financial statements can be attempted.

In my midterm report, I made the case that, given that the stated goal of financial reporting is to aid in making decisions about the reporting entity, uncertainties around current values cannot be ignored. Perhaps the most important contributor to these uncertainties is that many items in financial statements, though supposedly reflecting the proverbial "point in time", in fact represent "best guesses" as to the future. As I explain in more detail below, not only are these quantities uncertain, but their probability distributions can be sufficiently skewed that there is no single number that "fairly presents, in all material respects", the possibilities involved. Furthermore, the uncertainties in individual statement items are compounded when these uncertain quantities are used in subsequent calculations, most notably accounting ratios.

However, since my midterm report, I discovered a number of reasons why the accounting profession is unlikely to embrace an explicit characterization of financial statement items as random variables any time soon. Several studies in the latter years of the 20^{th} century, reviewed below, have shown that users of financial statements prefer to ignore the variability of numbers contained in them. One such study suggested a possible reason why: people simply don't want to deal with uncertainty, as is shown elegantly by the so-called Ellsberg paradox, as I describe below. Harris, *et. al.* also suggest, in both of their "worked examples", that random fluctuations in assets and liabilities can easily swamp operating income. Such a result could well compromise the credibility of any scheme to re-conceptualize financial statement items as random variables.

The above conclusions notwithstanding, my investigations led me to three areas where relatively little future research could lead to immediate benefits. First, there has been some effort to characterize the mathematical properties of accounting ratios as the quotient of two random variables, but an explicit calculation of distributions for these quotients, given plausible distributions for the numerator and denominator, await. Then there is the similarity between the computation of trial balances and debugging software. While a comprehensive theory of software bugs has defied researchers for decades, I present a useful "rule of thumb" below.

Finally, and most importantly, I concluded that the most immediate use of probabilities in financial statements is in "going concern" opinions. Accordingly, I prepared a draft of a paper making the case that auditors could and should include an explicit "going concern" probability in every opinion letter (Draft included as an appendix to this report.).

Why Financial Statement Items Need to be Treated as Random Variables

My midterm report proposes the terminology "materially stochastic" for a particular statement item if its variation is likely to have a material impact on the outcome of a decision concerning the reporting entity. The report presents examples of items that are likely to be materially stochastic, such as

- accounts receivable. As I note in the midterm report, "While current accounting practice requires various provisions for delayed payment and/or non-payment, nobody knows for sure who is going to pay and who isn't."
- percentage of completion revenues, where the activities that an entity must undertake to recognize contractually agreed upon revenues extend over multiple accounting periods. Here, GAAP requires that the fraction of the total revenue that can be recognized in the current period is the fraction of the project's total expenses that have been incurred during the current period. However, the total expenses of such a project may well not be known with certainty, especially for large construction projects that can run late.
- product returns, warranties, and other contingency sales. Here, the fraction of income that be recognized as revenue remains unknown until the sales become final.
- loan losses. Here, my midterm report presented a specific example in which a small, but unknown fraction of a portfolio of identical bank loans is expected to default. Because each borrower's default probability is determined partially by overall economic conditions, default events are correlated. The resulting distribution of losses to the portfolio is therefore far from the normal distribution obtained if losses were independent. In fact, this distribution is so skewed that its mode and median are only about 60% and 86%, respectively, of its mean, when parameters typical of the industry are used.

I also showed in the midterm report that there can be considerable bias in estimating the mean of the ratio $R = X/Y$ of two random variables, X and Y, by the ratio, μ_X/μ_Y, of their respective means, μ_X and μ_Y.

After the midterm report, I investigated the likely distribution of percentage of completion revenues, given that the numerator of the relevant fraction is known with certainty, but the denominator is a random variable. I was particularly interested in whether these completion revenues, when plausibly modeled as random variables, would have substantial skew, as such a skew would prevent In general, if $F_Y(y)$ is the cumulative distribution function of a random variable Y, $F_R(r)$ is the cumulative distribution function of the random variable $R = 1/Y$, and $r = 1/y$, then

$$\begin{aligned} F_Y(y) &= \Pr\{y < Y\} \\ &= \Pr\left\{\frac{1}{y} > \frac{1}{Y}\right\} \\ &= 1 - \Pr\left\{\frac{1}{y} < \frac{1}{Y}\right\} \\ &= 1 - \Pr\{r < R\} \\ &= 1 - F_R(r) \end{aligned}$$

It follows that $F_R(r) = 1 - F_Y(1/r)$. The density function of R is then

$$\begin{aligned} \frac{d}{dr} F_R(r) &= \frac{d}{dr}\left[1 - F_Y(1/r)\right] \\ &= -f_R(1/r)\frac{d}{dr}(1/r) \\ &= \frac{1}{r^2} f_R(1/r) \end{aligned}$$

Two of the distributions for project completion times (*i.e.* the denominator random variable, Y) that have been proposed in the literature are the beta distribution and the shifted lognormal (*e.g.* $Y = y_0 + X$, where X is lognormal)[1]. When Y has the beta distribution with positive real parameters α and β,

$$f_Y(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)},$$

[1] The use of the beta distribution for task completion estimates seems to have been enshrined in the project management literature since the U.S. Navy used it as part of the well known “program evaluation and review technique” (PERT) methodology in the 1950’s (Sherman, 2015.). See Trietsch, *et. al.* (2012) for a discussion of the more recent view that the shifted lognormal is preferable.

with $0 \leq y \leq 1$ and

$$B(\alpha,\beta)=\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\, dy,$$

then R will have the density

$$f_R(r)=\frac{(r-1)^{\beta-1}}{r^{\alpha+\beta+3}}$$

for $r > 1$, a density with a wide variety of skews, depending on the choices of α and β. Similarly, if Y has the shifted lognormal density

$$f_Y(y)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{[\ln(y-y_0)-\mu]^2}{2\sigma^2}\right\}\frac{1}{y-y_0},$$

defined for $y > y_0$,

$$f_R(r)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{\left[\ln\left(\frac{1-y_0 r}{r}\right)-\mu\right]^2}{2\sigma^2}\right\}\frac{1}{r(1-y_0 r)},$$

for $r < 1/y_0$. Again, $f_R(r)$ is a density with a wide variety of skews, depending on the choices of the parameters. It follows that their probability distributions can be sufficiently skewed that there is no single number that "fairly presents, in all material respects", the possibilities involved.

Also, since the midterm report, I briefly investigated the literature on the empirical distributions in accounting ratios. It seems that there is significant evidence (Podobnik, *et. al.*, 2011) that both the distributions of these ratios and the distributions of their quarterly changes can be characterized by so-called asymmetric Lévy distributions (of which the more familiar Cauchy distribution is a special case). Such distributions are known for their so-called "power law" tails; that is, if $f_X(x)$ is such a density, then it decays as $x^{-\alpha}$ as $x \rightarrow \infty$. Podobnik, *et. al.* find that $\alpha \approx 1$, implying that these distributions are extremely skewed (and that they have infinite variance).

Resistance to Treating Financial Statement Items as Random Variables

The thought that financial statement items should be treated as random variables seems to have a long history in the literature, after all. For example, Barkman (1977) made a proposal that line items, most notably for individual accounts receivable, are random variables. Barkman then traces through the effect of this in the sampling procedures used to verify accounts receivable totals, assuming that the individual account payments are beta distributed with parameters given by a practitioner. In a subsequent discussion piece (Barkman, 1978), he notes that an accounting entry is random because its true value will only become known in the future. In contrast, a measurement error "arises when repeated measurements of the same item yield different values … if an auditor were to evaluate an item many times during a given audit, the auditor would assign the same values … each time." Nevertheless, variability remains because of uncertainty about the future. In contrast, if only measurement error is relevant, repeated measurements that yielded the same results would lead to the conclusion that there is no variability. Barkman then gives the apt example of the accounting treatment of a basket of lottery tickets.

Other relevant studies include Johnson, *et. al.* (1981) and Frost and Tamura (1986). The former found great variability in error rates and significant departures from normality in audited and "book values" in 55 accounts receivable and 26 inventory audits. The latter noted that the skewness of the error distribution in individual accounts can invalidate the usual methodology for constructing confidence intervals. They study, in particular, additive errors, but they seem to argue that their results obtain for multiplicative errors, as well.

Nevertheless, a number of studies show that people seem to ignore confidence intervals when they are made available. For example, Oliver (1972) found that 123 professional bankers attending an in-service course "… did not significantly alter their hypothetical loan decisions …", depending on whether or not they were presented with financial statements equipped with confidence intervals around key statement items. Keys (1978) varied the details of Oliver's experimental design, but came to essentially the same conclusion. More recently, Ding *et. al.* (2009) found that 181 MBA students, when presented with confidence intervals around point estimates of fair value, actually did *worse* in making decisions about unlisted equity investing.

Another recent study, Du (2009) suggests, even by his decision to publish his study in *The Journal of Behavioral Finance*, that there is considerable psychology involved in the interpretation of financial information. For example, he cites Kahneman and Tversky (1979) to the effect that, in Du's words, "the gain or loss frame adopted by individuals directly affects their judgment and decisions." Du also reminds his readers of the all-important distinction between risk, in which there is sufficient information to estimate probabilities, and uncertainty, where there isn't. While Du's reading of the experimental evidence is that "individuals' attitudes toward ambiguity are neither universal nor uniform", he cites evidence that the uncertainty implied by confidence intervals is, itself, a significant disincentive to their use.

The existence of such "ambiguity aversion" has become known as the Ellsberg Paradox, due to the following example cited by Ellsberg (1961): Suppose an urn contains 30 red balls and 60 other balls that are either black or yellow. Suppose further that the balls are sufficiently well mixed that draws of balls of the different colors are independent events, but no further information is given. We are then offered the choice between being paid $100 if a red ball is drawn and being paid $100 if a black ball is drawn. Ellsberg reports that, not surprisingly, most people would opt to be paid if the red ball is drawn. Ellsberg then considers another choice, namely to be paid $100 (*i*) if *either* a red or yellow ball is drawn or (*ii*) if *either* a black or yellow ball is drawn. Again, most people prefer the choice with "more certainty", in this case the contingent payout if either the black or yellow ball is chosen, as this probability is known to be exactly 2/3. Yet any assignment of probabilities to drawing balls of different color, any assignment of utilities to the possible payouts, and any assignment of risk aversion leads to the conclusion that choice (*i*) should be preferred to choice (*ii*). It seems that the only way to explain people's actual preferences is to postulate such an uncertainty aversion.

Sure enough, per Foong, *et. al.* 2003, we find that everything does indeed depend on incentives – as well as the context in which they are offered. Specifically, Foong, *et. al.* asked 80 graduate business students, many of whom were already working, to make an investment choice among risky securities, paying half of them small sums if they met certain investment targets. They found that they were sufficiently motivated to use confidence interval information in making investment choices only when the monetary incentives were given. They caution, however, that the psychology of incentives is complex, in that people differentiate between boring tasks that require mechanical solution procedures and intrinsically interesting tasks that require heuristic, non-mechanical solution procedures.

Conclusions

My initial outline for this report had the headings "Why stochastic accounts are necessary", "Why stochastic accounts are problematic", and "What can be done in the near term anyway". The first two of these headings seem a fair summary of the above, especially given the complex psychology reported there. I turn now to the third heading, under which there are three sub-headings:

A Better Understanding of the Distributions of Individual Statement Items

Clearly, more could be done to characterize the probability distributions of individual statement items and, especially, the resulting ratios. From what I can tell, the probabilistic analysis given above would make a useful addition to the accounting literature, and I will looking into preparing the above for publication.

Exploring Parallels between Software Debugging and Trial Balance Reconciliation

The understanding that financial statement items are inherently uncertain allows us to ask about how to minimize that uncertainty. The preparation of financial statements is effectively the same kind of computational process as running a computer program. In an abstract sense, one could imagine that a single program would take as inputs all of the raw data (journal entries, etc.) and churn out a finished set of statements. In a more practical sense, the computations reflected in financial statements are, in fact, the outputs of one or more computer programs, along with some human intervention that could, in principle, be automated. In either case, the process is subject to errors, called "bugs" by software developers and simply "mistakes" by accountants.

But, in both cases, the process of correcting errors proceeds on a trial and error basis, with no expectation that every single error will be identified and corrected. Instead, the error elimination process terminates when the risk that remaining errors are material is determined (somehow) to be minimal. The determination in the case of software debugging is often specified as a minimum “mean time to failure”; the corresponding accounting metric is “likelihood of material misstatement”.

A rigorous model of the software debugging process seems to have not yet been formulated. The best that I have seen is based on the idea that each successive run of the program or preparation of trial balances produces an error with a probability proportional to at^{-b}, where t is the time since the beginning of the debugging process and a and b are positive constants (NIST/SEMATECH, 2012 states that “The Power Law (Duane) model has been very successful in modeling industrial reliability improvement data.” That). This gives a set of differential equations for the probability $P_k(t)$, the probability that k errors have been observed (and, presumably, fixed) by time t, that is a generalization to those for the Poisson process, namely

$$P_k(t+dt) = \left(1 - at^{-b}dt\right)P_{k+1}(t) + at^{-b}P_k(t)dt$$
$$\frac{dP_{k+1}(t)}{dt} = -at^{-b}P_{k+1}(t) + at^{-b}P_k(t)$$

A similar analysis leads to the probability that no errors have been observed by time t, namely, the solution to the differential equation

$$\frac{dP_0(t)}{dt} = -at^{-b}P_0(t)$$

whose solution is a version of the Weibull (stretched exponential) distribution, albeit with a parametrization that is different than the usual one. The advantage of this approach is that the parameters above can be easily estimated early in the “debugging” process, allowing extrapolation of the mean time to find the next error for a given level of “debugging”.

The use of explicit probabilities in “going concern” opinions

The one place probabilistic thinking could have an immediate impact on the accounting profession is in so-called “going concern” opinions. As is well known, auditors are currently required to say so in their opinion letter if there is “substantial doubt” as to whether the audited entity is, in fact, a going concern. Given the delicacy of such a disclosure (discussed more fully in the midterm report), it seems best to disclose the numerical results of a transparent default model. Accordingly, I attach, as an appendix, a draft of a paper making the case for replacing the binary standard with the requirement that every opinion letter include an estimate of a numerical probability of default, that the requisite models exist, and that their results would be viewed favorably by the courts.

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AUDITORS SHOULD INCLUDE A NUMERICAL "GOING CONCERN" PROBABILITY ESTIMATE IN THEIR OPINION LETTER[2]

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Introduction

The most important question that financial statements are supposed to answer about the reporting entity is whether or not it is a "going concern." Per FASB AU 341, an entity fails to be a going concern if is unable to

> *continue to meet its obligations as they become due without substantial disposition of assets outside the ordinary course of business, restructuring of debt, externally forced revisions of its operations, or similar actions.*

[2]I am grateful to the Institute of Management Accountants (IMA) Research Foundation for research support through its incubator grant program.

Aside from the immediate importance of the going concern question to stakeholders, the entire edifice of amortization and depreciation over multiple accounting periods makes little sense unless the entity expects to utilize all of its existing assets in the course of its operations. This will not be the case if, for example, the entity is in financial difficulties or if it has serious regulatory problems.

To form “a going concern” opinion, accountants and their auditors are not expected peer into the indefinite future. However, they are expected, also per FASB AU 341, to evaluate whether there is “substantial doubt about the entity’s ability to continue as a going concern for a reasonable period of time.” And, if auditors find that there is such doubt, they are *required* to say so in their audit opinion letter.

While the meaning of the phrase “reasonable period of time” is defined very clearly as “not to exceed one year from the date of the financial statements being audited,” there is no such clarity in the definition of the phrase “substantial doubt”. Indeed, Hu (2012) quotes a survey of what “substantial doubt” meant to stakeholders that included 45 auditors, 95 bank loan officers, 88 financial analysts, and 32 judges. The respective probability estimates had means (medians) of judges of 0.33 (0.30), of auditors 0.57 (0.51), of financial analysts 0.71 (0.70), and of bank loan officers 0.72 (0.75).

Wherever the “substantial doubt” bar is set though, GAAP still requires a binary decision: For any given entity, is there “substantial doubt” (whatever that means to the auditor) or not? This decision is a delicate one, especially since it is widely thought that doubt regarding the entity’s ability to continue as a going concern tends to be a self-fulfilling prophecy. Once doubt regarding the entity’s going concern status is raised in public, it is also widely thought that the entity’s suppliers will likely refuse any but cash sales, the entity’s existing employees will likely seek employment elsewhere, new employees will likely join the entity with reluctance, and any customers the entity has will likely consider alternative sourcing. Not only is management concerned about losing their own jobs in this scenario, but auditors are concerned about losing a client! Not surprisingly, therefore, a Bloomberg study quoted in Selling (2015) found that auditors failed to provide a “going concern” qualification for 54% of the 673 largest bankruptcies between 1996 and 2002, the height of the internet bubble.

Public concern about the failure of audits to uncover going concern issues has reached the point where the United States Senate felt the need to hold a hearing on the matter (http://www.banking.senate.gov/public/index.cfm?FuseAction=Hearings.Hearing&Hearing_ID=0f533e5b-dc43-4fc2-a415-5df2ae8806da). The testimony from that hearing included the following statement from James R. Doty, Chair of the Public Company Accounting Oversight Board (PCAOB):

Nothing less than a fundamental reassessment of the role of auditing in public company financial reporting can begin to accomplish what is needed after the three auditing Armageddons in three decades: the S&L crisis, Enron et al, and the 2008 financial crisis.

The FASB has attempted to address this problem via a 2013 exposure draft requiring *management* to report going concern issues. If this proposal becomes finalized, it will put management in a difficult position: If management truly believes that the company is not a going concern, then either management should be replaced by people who have more faith in the entity's viability or management should immediately liquidate the entity, as any delay will likely be costly to stakeholders. Also, management may well be required to make this disclosure based on non-public information, some of which is extremely sensitive. Consider, for example, pending litigation, which is plagued with uncertainties. Defendants often expect to have to make a large settlement or pay a large judgment in an upcoming lawsuit, only to find that the plaintiff is willing to settle the suit for significantly less than the defendant expected. The proposed going concern directive would force management to tip its hand if such a lawsuit was pending at the time that financial statements are being prepared.

The primary purpose of this paper is to provide an alternative solution to the going concern problem. I argue that, rather than being required to make a binary "going concern" decision, auditors should be required to publish an explicit probability that the audited entity will default in the upcoming year. Since this probability is replacing a completely subjective, binary assessment of the likelihood of upcoming difficulties, a model that estimates – or even bounds – this likelihood to the nearest 10% is an improvement. I note in passing that the twin problems of the binary going concern decision – that a binary concern qualification is issued when it shouldn't be (Type I error) and that a binary concern qualification isn't issued when it should be (Type II error) – disappear with an explicit probabilistic estimate.

As we will see, there is a large and growing literature on the estimation of these default probabilities, reflecting the three substantially different quantitative approaches to default prediction. Historically, the first default prediction models consisted of appropriately weighted averages of various accounting ratios (See, for example, Bellovary, *et. al*. 2007, for a review some 135 models, mostly of this type[3].). Perhaps the best known of the models in Bellovary, *et. al.* 's inventory is the widely used z-score model of Altman (1968). Though Altman's z-score, like many other, similar models included in Bellovary, *et. al.*'s review, is currently used in a binary "default/no default" prediction, the underlying assumption is that lower scores imply a greater likelihood of default. It follows that such scores could easily be mapped into

[3] Bellovary *et. al.* also mention bankruptcy prediction using neural networks. While they report that a number of such models seem to perform admirably, neural nets are well known for their lack of transparency. Hence, I do not think they are suitable for the present application.

default probabilities, especially if Altman's discriminant analysis is replaced by the related technique of probit analysis (See, for example, the discussion of "models of binary choice" in Greene, 1993.).

As I will discuss more fully below, various authors have expressed reservations about models based on inherently "backward looking" accounting ratios, as opposed to more recent bankruptcy that make use of "forward looking" financial market price data. In fact, Altman's z-score also includes the ratio of market value of total equities to the book value of total liabilities, and I argue below that it is this feature that accounts for much of the model's success. I am also reluctant to advocate the use of models based only on accounting ratios for going concern probabilities for another reason: the going concern probability would be embedded in the same financial statements that are used as inputs for the probability calculation. Thus, a probability that is derived from these ratios alone adds no value.

However, before embarking on a detailed discussion of how – and how well – these probabilities can be estimated, we address a more immediate concern. Auditors facing the prospect of legal liability for whatever statements they make in opinion letters will, no doubt, wonder about how courts would view the inclusion of an explicit default probability in such letters. Accordingly, the next section of this paper reviews some recent decisions by the federal courts regarding what is effectively a going concern question in litigation concerning so-called "constructive fraud" in failed leveraged buy-outs (LBO's). There, I report growing deference to going concern status inferred from financial market price data, rather than expert opinion.

Having made a case that financial market data can and should be used for estimating going concern probabilities, I then consider the question of how to actually do so in some detail. Given equity price series, so-called structural models build upon Merton's insight that corporate equity is, in fact, a call option on the corporation's assets, so that the probability of going concern difficulties is the probability that corporate assets decline below corporate liabilities. Included in this discussion is a review of the formidable difficulties in translating Merton's idea into a model that embraces the complexities of default prediction for real corporations. I include this review for two reasons: (*i*) I want to make clear that a model that addresses these difficulties lacks the transparency desirable for a going concern calculation, and (*ii*) previous attempts at comparing default predictions from structural models with other models are invalidated by their failure to address these difficulties. Thus, a second contribution of this paper is a critical assessment of these previous attempts.

Alternatively, so-called hazard rate models infer the probability of going concern difficulties from given bond price series, or, better still, historical credit default swap (CDS) spreads, by equating the yield spread over the risk free rate to the expected cost of default. Probabilities thus computed are, in many cases, the easiest to access. Credit default swaps for many large corporations are so actively traded that the implied default probabilities are quoted directly on Bloomberg terminals. While we will see below that such probabilities are not the actual probabilities of default, but so-called "risk neutral probabilities", they are, nevertheless, always upper bounds for the actual default probabilities. In many cases, though, such an upper bound is sufficient: the upper bound, so often less than one percent per annum, implies that the actual probabilities are even smaller. I conclude that the same case that Simkovic and Kaminetzky (2011)), hereafter "S&K", make for a simple hazard rate model for resolving LBO "constructive fraud" cases can also be made here. They write,

> *The techniques we develop and present here are designed to be simple. Although professional fixed income traders—who depend for their profits on fractions of a percent on every trade …—may use more sophisticated techniques, extracting market implied probabilities of default for our purposes does not require the same degree of precision. … In the legal context, simplicity is a virtue.*

I would argue that simplicity (and transparency) are virtues in the present case, as well.

On a proposal to use contemporaneous financial market data to determine constructive fraud

S&K propose the use of explicit default probabilities in cases involving so-called "constructive fraud" in leveraged buy-outs (LBO's). Their paper concerns the "what-did-they-know" and "when-did-they-know-it" questions surrounding so-called fraudulent transfer on the part of the LBO orchestrator. Since similar questions are likely to arise in litigation surrounding going concern opinions, much of S&K's argument applies in the present case as well. Hence, I present some details of the S&K paper.

Recall that an LBO consists of a buy-out of the existing shareholders of a target firm, a shifting of the resulting debt obligation to the target firm, and a servicing of the debt with the target firm's subsequent profits. In some of these transactions, the resulting heavy debt burden on the target makes its subsequent default all but inevitable. The legal doctrine of constructive fraud asserts that the target firm's creditors can then sue to void the payment of funds to bought-out shareholders on the grounds that the payment was a fraudulent transfer, in that it rendered the target firm inadequately capitalized. To win such lawsuits, the plaintiff must establish that the orchestrators of the buy-out knew or should have known of the problem *at the time of the transaction*.

S&K characterize current litigation in this area as a "battle of the experts", in which sharply contradictory estimates of the post-payout firm's value and liquidity are exchanged. While the basis for such estimates are the generally accepted tools of solvency analysis and valuation, S&K observe that these tools are often misused to suit the purposes of their users, who, after all, are paid by the respective parties to the lawsuit. S&K are especially concerned with the problem of "hindsight bias", as it is all too tempting to blame *any* subsequent financial problems of the target firm on the LBO, even if such problems could not possibly have been foreseen. Providing case citations that are suppressed below, they write

> *Foreseeability is determined on a case-by-case basis, but such an ad hoc approach to justice provides little guidance to counterparties structuring transactions. In many cases, courts have reached seemingly inconsistent determinations about whether a particular type of business setback is foreseeable. Low-cost competition is apparently foreseeable in the automotive industry, but not in mobile communications. Loss of revenue is apparently foreseeable if it is due to the loss of a key customer, but is not foreseeable if it is due to the loss of a key employee. Financial crises are apparently not foreseeable if they are due to defaults by poor formerly communist countries, but financial crises are foreseeable if they are due to defaults by poor subprime mortgage borrowers. The failure to achieve post-merger synergies might be foreseeable or might not be foreseeable, but judicial opinion on the matter certainly is not.*

In other words, the LBO orchestrator and the court, in reviewing the LBO orchestrator's decision, must effectively face the same issues as an auditor making a going concern decision.

S&K then observe that the Supreme Court of the United States has "has long embraced the belief, widely shared by many Anglo-American economists, that well-regulated financial markets effectively process available information and thereby fairly and appropriately value securities." Indeed, in *Basic v. Levinson*, 485 U.S. 224, 245-46 (1988), the Court opined that "[T]he market . . . ideally transmits information to the investor in the processed form of a market price." This same opinion created the so-called "fraud in the market" economic theory. Per Nili (2014), the fraud in the market theory holds that misrepresentations concerning public companies constitute fraud if (1) the alleged misrepresentations were publicly known, (2) they were material, (3) the stock was traded in an efficient market, and (4) the plaintiff traded in the stock in the relevant period."

Since *Basic*, the courts have repeatedly affirmed S&K's statement that "the … Court has accepted the semi-strong form of the Efficient Market Hypothesis". S&K cite two cases, *VFB v Campbell's Soup Co*. and *in Re Iridium Operating LLC* (In re Iridium Operating LLC, 373 B.R. 283, 291, 352 (Bankr. S.D.N.Y. 2007). in which lower federal courts went so far as to substitute market prices for expert opinion. Since both of these cases show the degree to which courts have already demonstrated willingness to defer to financial markets, I now discuss them in some detail.

In *VFB*, Campbell's Soup Co. received $500 million in cash from a newly formed entity, Vlasic Foods International ("VFI") that assumed the $500 million as debt in exchange for some of Campbell's underperforming product lines. Though this transaction resembled an LBO, VFI shares traded in public equity markets – at least until VFI, subsequently reorganized as VFB, filed for bankruptcy three years later. S&K discuss the case as follows (with case citations deleted):

> *For two years before the spin-off, Campbell used a variety of dubious accounting techniques to improve the reported finances of the division that would become VFI without actually improving its long-term prospects. These manipulations appear to have successfully misled both the securities markets and the banks that extended credit to finance the spin-off transaction.*
>
> *However, shortly after the spin-off, VFI's "inflated sales and earnings figures quickly corrected themselves." The market presumably processed this new, more accurate information about VFI's past performance and future prospects, but VFI's market capitalization remained above $1.1 billion and the company was able to raise $200 million in new unsecured debt. The [Delaware District Court] Court interpreted equity market prices and bond market receptivity as a judgment by the capital markets that VFI was solvent as of the date of the spin-off, and that the spin-off therefore could not be avoided as a fraudulent transfer.*
>
> *The Court suggested that the period at which the debtor became insolvent could be determined based on the time when the debtor's bonds began trading below par value. It should be noted that at the time VFI filed bankruptcy, bond markets were generally over-the-counter markets with very little public disclosure of transaction pricing or trade volumes—unlike the liquid, transparent, exchange-traded stock markets discussed by the Supreme Court in Basic v. Levinson—but the Court nevertheless deferred to bond market prices.*

The Third Circuit Court of Appeals upheld this decision, noting with approval that, in its words, "[B]asically, the district court regarded the hired expert valuations as a side-show to the disinterested evidence … of the most efficient capital markets in the world [*i.e.* the U.S. stock market]."

S&K comment in the second case, *In Re Iridium, LLC*, that

> *Iridium is noteworthy because [the U.S. Bankruptcy Court for the Southern District of New York] resisted the temptation to second-guess market participants' contemporaneous judgments, even though the market was so bad at predicting the future*

> *performance of the debtor that the market's valuation in this instance seems almost absurd—at least with the benefit of hindsight.*

The facts of the case are as follows (S&K and Finkelstein and Sanford, 2000): In the early 1990's, Motorola spun off its partially built satellite-based portable telephone system as Iridium, Inc., which paid more than $3.5 billion to Motorola for the system. Iridium completed development and made the system available to the public in 1998, attracting considerable Wall Street acclaim and, well, extra-terrestrial valuations. Iridium filed for bankruptcy in 1999, having failed to compete against the vastly superior technology of the cell phone.

Iridium investors lost the subsequent constructive fraud suit against Motorola. The Court opined that

> *The fact that Iridium failed in such a spectacular fashion stands out as a disturbing counterpoint to the market's optimistic predictions of present and future value for Iridium, but in the end, the market evidence could not be denied. The capital markets synthesized and distilled what all the smart people of the era knew or believed to be true about Iridium.*

In other words, the Court was prepared to defer to the wisdom of the markets, even when, in hindsight, they weren't so wise, after all.

S&K summarize the above by writing "Market-implied probabilities of default can assist courts in deciding *any* controversy that requires a judicial determination of corporate solvency…" (Italics mine).

Models of Default

As noted in the introduction, default probability models can be based on either accounting ratios, equity prices, or credit market (bond and credit default) prices. In this section, I review each of these approaches. First, however, some general remarks on exactly what needs to be predicted and what the models offer.

On What Needs to be Predicted and What the Models Offer

The first task of using a model of the kind described above is to make sure that the predictions that the model attempts to make coincide with the failure to be a going concern, as defined above. Part of the difficulty in doing so arises with inconsistencies in the FASB definition of going concern. This definition implies that an entity has failed to be a going concern if it will need to make a minimal restructuring of a single debt payment in the coming year, even if such a restructuring does not cause the liquidation of assets that would delegitimize depreciation and amortization over multiple accounting periods. Yet even a "Chapter 11" bankruptcy filing need not cause such a liquidation under the U.S. bankruptcy code, as such filings usually include a plan by which the entity remains "alive" and pays its creditors over time (US Courts, 2015).

In fact, bankruptcy itself, though apparently well defined in black and white accounting terms as "liabilities exceeding assets", has, in practice, a subjective component. Recall that bankruptcy is a decision made by a court, pending a filing either by an entity that can't meet its obligations or by the entity's creditors. In either case, the decision is based, not just on cash-in-hand and strict payment deadlines, but on creditor flexibility or lack thereof and on the entity's immediate future prospects. Furthermore, in the event of liquidation, courts do not always recognize the strict seniority of debt that seems to be implied by the liquidating entity's capital structure.

With the above as caveats, I propose to identify the probability of failure to be a going concern with the more widely studied probability of default, but care must be taken in estimating these probabilities from those implied by actively traded market instruments.

A naïve approach to this inference problem would back out these probabilities by assuming that the current value of the instrument is today's expected value of its future payouts, given what is known today. However, the fact that investors are, in general, risk averse needs to be considered, as can be seen from the following example: Most investors would expect to pay less than $40 for a chance to receive a payment whose present value is $100, but with only a 40% probability (There is, after all, no sense in entering a transaction in which, on average, the investor expects to break even.). Yet an observed price of, say, $35 for such a chance would lead, via the naïve methodology proposed above, to the erroneous conclusion that the probability of receiving the $100 was 35% and not 40%. The problem is that all that can be observed is the expected utility of the payout, *i.e.* the product of the probability of the payout and the utility of the payout to the investor if it is made, and the utility of a contingent payment to most investors is reduced from its nominal amount.

Modern options theory solves the problem of unknown investor utilities by assuming that the utility of a contingent claim is its nominal amount, but expected utilities for contingent claims are to be computed using a set of "risk adjusted" probabilities, rather than the actual ones (See, for example, Neftci, 2000 for an elementary account). Thanks to the risk adjustment, expectations of future payouts computed with respect to this distribution *will* reflect the collective risk aversion of the market. Thus, it is these probabilities – and not the corresponding "actual" probabilities – that will be implied by observed market prices. Clearly, the only way that expected utilities computed using this scheme can match observed market prices is if the risk adjusted probabilities of receiving future cashflows are strictly *less* than the corresponding actual probabilities. Since the failure to receive one or more of these cashflows is, by definition, a default, it follows that the corresponding risk adjusted probabilities of failing to receive these cashflows, *i.e.* the probabilities of default, are strictly greater than the corresponding actual probabilities of default.

Bellovary *et. al.* (2007) made the following observations about the assessment of bankruptcy prediction models that are useful for present purposes:

- Different models are "tuned" to different kinds of entities. Altman's 1968 model was, for example, tuned for use with manufacturing firms.
- Models should be evaluated based on their performance with a "hold-out sample", *i.e.* a set of entities entirely distinct from the "training sample" of entities used to estimate model parameters.

With the above in mind, I now consider specifics of the three bankruptcy prediction methodologies described above.

Models Based on Accounting Ratios

Bellovary *et. al.* report that several multivariate discriminant analysis models in their collection were able to make the correct "bankruptcy/no bankruptcy" decision for each future bankruptcy in their hold-out samples. However, the same could not be said for models that tried to make the same binary decision using probit analysis (*i.e.* estimating a bankruptcy probability and then declaring an entity as a future bankrupt if its probability exceeds ½.). In fact, the model that they found to accurately identify the most future bankrupts, a model trained on Swedish manufacturing firms, did so only 84% of the time. This suggests that the ranking of firms in order of bankruptcy likelihood, which is implicit in assigning a default probability to each firm, is more difficult than simply making the "bankruptcy/no bankruptcy" decision.

Agarwal and Taffler (2008) suggest that this problem is only one of many with accounting based models. They observe that ratio analysis can identify when a firm is on the threshold of bankruptcy, but it cannot assess the likelihood that it will cross that threshold (See the discussion of structural models below for more details.). Agarwal and Taffler are also concerned that true asset values might (and usually are) quite different from the historical book values appearing in financial ratios and that "accounting numbers are subject to manipulation by management" (a temptation that presumably becomes stronger as bankruptcy approaches).

Heilegeist, *et. al.* (2004) takes the criticism of accounting based models a step further, pointing out that financial statements are prepared on a going concern basis, which explicitly rules out the possibility of bankruptcy (!).

Structural Models

The impetus for structural models of default was the observation in Merton (1974) that the value, E, of a firm's equity is effectively a call option on the assets of the firm, with the strike price set at the face value, L, of the firm's liabilities. This observation follows from the accounting definition of equity as the maximum of the difference between the assets of the firm and the face value of its debt, *i.e*, $E = A – L$, if the firm is solvent and zero otherwise. Note that such an option can have positive value even if the value of the assets is near, or even at the value of the liabilities, because there is the possibility that the firm's prospects will improve.

Merton and many subsequent authors (See Bharath and Shumway, 2004 for a widely cited example) assumed the simplest kind of corporate structure: a corporation that has as its only liability a discount bond (*i.e.* a bond with interest and principal payable in full at maturity and no intermediate interest payments). At the maturity of the bond, the corporation is either able to pay its bondholders in full from assets that exceed liabilities or its entire asset base, if assets do not exceed liabilities. The latter case is, of course, a default.

Given the above simplification of a real firm's capital structure, and given the assumption that changes in asset values are lognormally distributed[4], the relevant call option is identical to a European call option on the assets. Merton was therefore able to use the celebrated Black Scholes Merton option pricing theory (see, for example, McDonald, 2009, for a textbook introduction) to deduce the risk adjusted probability of default, given the current asset and liability values, the current riskless interest rate, and the volatility of asset prices. However, real corporations must make ongoing payments on their debt and other liabilities, so there is an ongoing possibility of default throughout the foreseeable future. This significantly changes

[4] The standard Black Scholes Merton theory, as described in, for example McDonald (2009), assumes that equity, rather than asset price changes are lognormally distributed. However, it is a "stylized fact" that the volatilities of equity options vary inversely with strike, precisely what would be expected if assets, rather than equities, were lognormally distributed.

the nature of the "option to default," from a standard European call option to a perpetual "down and out" option[5].

Undeterred by the above complexities, the KMV Corporation created and marketed a service that provided (and continues to provide, even after their purchase by Moody's Investor Service in 2002) default probabilities for profit making corporations[6] using the above ideas. In doing so, they had to address several important practical issues, as follows (Crosbie, *et. al.* 2000, Kealhoffer, 2003):

- The Merton model as stated in Merton (1974) will yield a risk adjusted default probability, rather than the desired actual default probability. They do so by first computing a so-called "distance to default", discussed in more detail below.

- Neither current asset values nor current asset volatilities are directly observable. KMV determined these quantities implicitly from observed equity prices and equity volatilities by the simultaneous solution of the nonlinear equations

  $$\sigma_A^2 = \sigma_E^2 \left(\frac{\partial A}{\partial E} \right)^2 \left(\frac{E}{A} \right)^2$$

  and

  $$E(A, \sigma_A) = E_{\text{observed}},$$

  where σ_A and σ_E are the respective volatilities of the assets and the equity, $E(A, \sigma_A)$ is the value of the equity, expressed as a function of the assets and asset volatility, and E_{observed} is the observed value of the equity.

- Most corporations have both short and long term liabilities, some of which can be deferred. KMV set the "default point", *P*, somewhat arbitrarily as the sum of the long term liabilities and half of the short term liabilities.

- The risk free rate used in the calculations should be somewhat higher than the short term treasury rate for the relevant currency because treasury markets tend to be distorted by regulations.

- Corporate asset fluctuations, like most financial price fluctuations have "fat tails", so large percentage fluctuations are observed more often than would be the case if asset prices were

[5] This option ceases to exist if the underlying price (in this case, the price of the firm's assets) falls below the barrier (in this case, the value of the liabilities), but it exists in perpetuity otherwise.

[6] The original KMV model was designed for public companies, and thus companies with a publically available equity price. They have since created a "private firm" model.

lognormal. KMV dealt with that problem by computing a so-called "distance to default", which is the negative return that the corporation's assets would have to earn to cross the default point within a given time, scaled by σ_A. The distance to default is thus an ordinal ranking of firms' proximity to default. KMV then assumes that this ranking doesn't change whether they are ranked by risk adjusted or actual probability of default. This assumption justifies KMV's next step: the use of empirical default data to determine the historical default frequency of corporations with the same distance to default. KMV then reports the result as its so-called "expected default frequency", or EDF, as an actual, rather than"risk adjusted" probability of default.

I present the above details of the KMV model because they are relevant to the practical estimation of going concern probabilities for two reasons. First, the above details make clear that a careful estimation of default probabilities is a non-trivial proposition. Not only does it require a mastery of the advanced mathematics of option pricing theory, but it also requires extensive data collection to determine default points, distances to default, and empirical EDF's. Furthermore, the model described above clearly requires accurate historical equity prices as input, implicitly requiring the target firm to be a public company. The task of mapping a private firm of interest to one or more suitable public firms, as would be required to make the model generally applicable, as KMV did is an even more substantial data collection exercise.

Then there's the question of how well the model actually works. Academic studies, such as Heilegeist, *et. al.*, Barath and Shumway (2004), Barath and Shumway (2008), and, more recently, Camara, *et. al.*(2011) claim to perform empirical tests. Heilegeist, *et. al.* find that their particular version of the basic Merton model performs about as well as a commercial version Altman's z-score model, with parameters re-estimated with more recent data. The others conclude that a model "based on … [the] model developed by the KMV Corporation" (in Barath and Shumway's words) do considerably better than chance", but worse than the authors' particular variant of the basic Merton model. However, both sets of authors report that they did not have access to the proprietary database that KMV uses to translate distances to default into EDF's and therefore cannot claim to be able to reproduce the actual KMV model. Instead, their tests are based on the simplified, lognormal model given above.

However, the portion of the distribution of asset fluctuations relevant to default prediction (*i.e.* the left tail) is precisely the portion of the actual distribution least likely to be well approximated by the lognormal distribution. Errors in estimating default probabilities by using lognormal asset fluctuations, rather than a more appropriate empirical distribution, may also be amplified by modeling equity as a European option on assets, rather than the more correct perpetual knockout barrier option.

In a later version of their 2004 paper (Barath and Shumway, 2008), they state that "we do not intend to imply that we are using exactly the same algorithm that Moody's KMV … uses to compute distance to default." They argue that the differences in the results are, nevertheless, immaterial. They base this argument on a direct comparison between their calculations and EDF's that Moody's KMV published in *CFO Magazine* for the largest 100 debt issuers (Fink, 2003). For those 80 firms for which Bharath and Shumway had data, they report a rank correlation of 79%. However, they do not report on the nature of the rankings that differed, so we cannot infer from their work whether these ranking differences – and the resulting default probabilities – were significantly larger for issuers nearer the default point.

I draw the following conclusions from the above:

1. If a leading accounting based model performs as well as an inferior version of a structural model, a properly implemented version of the structural model (i.e. the KMV model) should perform notably better.
2. Claims regarding the KMV model's default prediction efficacy versus the other structural models reported in the above cited references remain to be proven.
3. Given the first two conclusions, the KMV model is likely the most accurate source of actual, as opposed to risk adjusted default probabilities that are available. However, the difficulties that seem to have prevented the above researchers from reproducing the KMV model exactly render the KMV model far too opaque for producing going concern probabilities.

Reduced Form Models

Default probabilities can also be inferred from market prices of either bonds or credit default swaps (CDS). The basic idea is the same for both, but each has advantages and disadvantages. Recall from the above that, under fairly mild assumptions, a consistent set of "risk adjusted" probabilities can be found such that market prices of financial instruments correspond to the expected value of their future cashflows. Thus, it is, in principle, possible to "back out" the risk adjusted probabilities corresponding to given market prices. It is the difficulty of going from "in principle" to practice that creates the advantages and disadvantages of using each of the two fixed income asset classes mentioned above. Fortunately, these difficulties are mitigated by the fact that only a rough estimate is required for present purposes.

Consider, first, the task of inferring default probabilities from the observed yield on default risky debt. This is possible because of the no arbitrage condition that the expected future value of the risky debt after one year should also be that of risk free debt of the same maturity. This condition should obtain in an efficient market in which investors buy and sell debt until they perceive the expected payouts to be equal.

For the purposes of this rough estimate, assume that default can only occur at the end of the year. Now, let r be the risk free simple interest rate for one year (*i.e.* the interest rate payable on an instrument where there is no risk of default), let y be the corresponding rate for default risky debt, let p^* be the risk adjusted probability of default, and let l be the "loss severity", or the fraction of the principal lost, given default. If a dollar of principal invested in the risk free debt, the future value of the investment is $1 + r$. Alternatively, if this dollar is invested in the the risky debt, the expected future value is the sum of the expected value in the non-default state, namely $(1 - p^*)(1 + y)$ and the expected value in the default state, when the amount $(1 - l)$ is recovered for each dollar lost. The expected value over both default and non-default states is thus $(1 - p^*)(1 + y) + p^*(1 + y)(1 - l)$. Equating the expected future values of the risk free and risky debt, it follows that

$$(1 + r) = (1 - p^*)(1 + y) + p^*(1 + y)(1 - l),$$

or

$$\begin{aligned} p^* &= \frac{1}{l}\left(\frac{y - r}{1 + y}\right) \\ &\approx \frac{1}{l}(y - r), \end{aligned}$$

since y is usually less than 10%.

The most obvious practical difficulty in using the above formula is the availability of the data. Loss severity, l, cannot be directly observed in the market. However, Moody's Investors Service publishes historical recovery rates (*i.e.* $1 - l$) of corporate bonds and loans from 1920 to the present, and industry professionals use this and similar studies to justify a loss severity of 60%.

Yield data also deserves comment. Bond prices, and thus their implied yields have become much more readily available in the last decade, due to FINRA's Trade Reporting and Compliance Engine (TRACE), a system to disseminate bond transaction and price data at no charge to the public. However, not all corporate bonds trade regularly. Also, care must be taken that bonds used for the going concern calculation are free of call or put features or other contingent claims (See, for example, Sundaresan, 2009, for a text book discussion.). If there is no secondary market in the relevant entity's debt, just about any entity can get indicative yield quotes from prospective lenders.

As noted above, the determination of the risk free rate is not obvious. The conventional wisdom is that the risk free rate for U.S. dollar denominated debt for a given maturity is the yield on U.S. treasury debt of the same maturity. However, a number of authors, such as Hull, *et. al.* (2005) and Kealhoffer (2003), believe that treasury yields are artificially depressed below other investments with very low credit risk. Hull, *et. al.* (2005) cites three ways that U.S. treasury debt receives favorable treatment by U.S. regulators:

1. Banks and other financial institutions are required to hold treasury debt to fulfill regulatory requirements. This artificially inflates prices, thus lowering yields.
2. Banks are required to hold considerably less capital to support investments in treasury debt than debt that is only marginally more credit risky.
3. Interest on U.S. treasury debt is not taxed at the state level.

There has been considerable academic study of the empirical spread between corporate bond yields and corresponding treasury debt yields (See, for example, Fons (1987), Elton, *et. al.* (2001), Hull, *et. al.* (2005), Longstaff, *et. al.* (2005)). Not surprisingly, given the risk adjustment argument given earlier, these studies consistently find that the empirical spread overestimates default experience. What is more surprising is the extent of this overestimation, an effect that has sometimes "the credit spread puzzle." Hull (2013) suggests that the effect is so large that "[b]ond traders may be allowing for depression scenarios much worse than anything seen during the period covered by historical data."

Many reasons have been proposed to account for the credit spread puzzle. While any depression of treasury yields below a hypothetical "market" risk free rate could be one of them, Longstaff, *et. al.* (2005) provide empirical results (albeit over a short time period) to dispute this point. They emphasize, instead, the importance of liquidity, as corporate bonds can have significant bid offer spreads. Hull (2013) cites more recent research, such as Dick-Nielson, *et. al.* (2012), that downplays the importance of liquidity. Instead, Hull (2013) notes that default risk is not easy for a bond portfolio manager to "diversify away".

Alternatively, the required default probabilities could be estimated using credit default swap (CDS) data. Such data is less readily available than bond market data, and, when it is available, it pertains to only the largest enterprises. However, as will be discussed below, there are several reasons why it should be used in addition to, or even instead of bond market data.

A CDS contract has similar economics to an insurance policy that all principal and interest payments on a given debt will be paid in full. A CDS is therefore a bet on whether a given "reference entity" will default on a particular debt. Like an insurance policy, the CDS holder ("protection buyer") typically pays a periodic premium to the insurer ("protection seller"). In return, the protection buyer receives, in the event of a default by the reference entity, a payment from the protection seller, based on losses that would be incurred by a hypothetical investor holding a given "notional" amount of the reference entity's debt. It should be noted, however, that a CDS contract is not, technically, an insurance policy: anyone can enter into a CDS, regardless of whether they have an "insurable interest" (that is, regardless of whether they would sustain losses if the debt defaults). Another difference with insurance, crucial for present purposes, is the fact that CDS contracts, unlike insurance contracts, are actively traded.

Two additional facts about the economics of CDS contracts make them attractive proxies for "true" bond yields. First, the debt insured by a CDS can be replicated (more or less[7]) by the CDS holder, simply by buying default free debt of the same maturity as the contract. If the annual CDS premium is s, therefore, the corresponding risky debt yield should be $r + s$. The same economic argument as above leads to the conclusion that

$$\begin{aligned} p^* &= \frac{1}{l}\left(\frac{s}{1+r+s}\right) \\ &\approx \frac{s}{l}. \end{aligned}$$

Both Longstaff, *et. al.* (2005) and S&K view the CDS market as having less "market friction" than the bond market, making for former a preferred source for estimates of p^*. Longstaff, *et. al.* (2005) explains that CDS spreads are far less sensitive to liquidity effects because CDS are not issued in fixed amounts, as are bonds: If buyers generate sufficient demand, sellers will simply create more CDS contracts. Also, it is easy to unwind a CDS contract by entering into an equal and opposite contract, as it is generally just as easy to sell protection as to buy it. In contrast, it can be difficult and costly to bet a bond will fall in value via a short sale.

S&K cite several studies showing that CDS markets anticipate negative credit rating agency actions (reviews for downgrade, negative outlooks, and downgrades) earlier than either the stock or bond markets. These studies also suggest why: CDS market participants are largely sophisticated financial institutions that may also be secured creditors or have other reasons to have non-public information about a debtor.

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[77] The caveat is that the CDS holder is subject, not only to the default risk of the reference entity, but also to the CDS protection seller. Since most protection sellers are highly rated financial institutions, this latter risk is generally viewed as negligible.

Summary and Conclusion

When audtors have "substantial doubt" as to the ability of the audited entity to remain a going concern, FASB AU 341 requires them to say so in their audit opinion letter. This places auditors in a difficult situation, as such a statement is all too likely to become a self-fulfilling prophesy. This paper argued that the binary judgment of FASB AU 341 should be replaced by a numerical probability that the audited entity will fail to be a going concern within the next year. I present evidence that this probability can be estimated by a simple and transparent formula from bond yield spreads or credit default swap spreads, after showing that courts are coming to view such calculations favorably in a different, but related context. In the process, I explained why default probability estimates from bonds or credit default swaps are to be preferred over methods using accounting ratios and equity market data.

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