

The Right Balance? Using a quantum annealing computer for your portfolio

September 21, 2020

D-Wave Systems Annealer & Classical Methods

PICKING US EQUITY PORTFOLIOS CLASSICALLY, AND WITH A QUANTUM ANNEALING COMPUTER

FS CLUB (LONDON)
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CHICAGO & BALTIMORE, USA

STOCK PORTFOLIO OPTIMIZATION

- **We reformulate the Sharpe ratio (buy & hold strategy using 12 months historical data) to run on D-Wave 2000Q 2,048 qubit quantum annealing computer (2017)**
- **We scale problem to 40, 60, and max 64 stocks on the D-Wave quantum annealer**
- **We use 5 classical solvers, and the D-Wave, to find efficient portfolios in < 1 minute per method**
- **The 'ideal' portfolios published in July 2020 and August 2020 under-performed the market and benchmarks through September 16, 2020**
- **We scale the classical solvers up to 1,855 stocks (all NYSE common stocks) in ~1 hour overall**
- **Net-Net: Investors can use either classical methods or quantum annealing computers to build efficient equity portfolios out of 64 stocks or less**

We continue our research (adding simulated bifurcation model & quantum walks on graphs)

NEW FORMULATION

- **Quantum annealing computers cannot divide, but they can subtract (linear algebra)**
- **Sharpe Ratio (and CQR) divides, while Chicago Quantum Net Score subtracts**
- **The CQNS formulation works on a quantum annealer and other classical methods**

This work is structured as follows: In §2 we begin our exploration with the Sharpe ratio

$$S_a(w) = \frac{w\beta\mathbb{E}[R_a - R_b] + R_b}{\sigma_a} \quad (1)$$

Where β is the ratio of Covariance of a portfolio with the market over the variance of the entire market [3], R_a is the return of the collection of assets, R_b is the risk free return, and σ_a is the standard deviation of the collection of assets, and w is a vector of weights for assets in our portfolio.

We can also see the Sharpe ratio in matrix form as

$$S_a(w) = \frac{w\beta\mathbb{E}[R_a - R_b] + R_b}{[w^t\text{Cov}_{ij}w]^{1/2}} \quad (2)$$

From here we develop the Chicago Quantum Ratio (CQR)

$$\text{CQR}_a(w) = \frac{w \cdot \text{Cov}_{im}}{\sigma_a} \quad (3)$$

Buy and hold strategy based on 12 months of historic data

- BETA (0,2.5)
- Continuously traded US-listed stocks
- Can 'tile together' independent, random samples
- Market and risk-free index data (floors & ceilings)

This, however causes a different set of mathematical problems in formulating a consistent quadratic form. Finally we settle on the Chicago Quantum Net Score (CQNS) which is given by

$$\text{CQNS}(w; \alpha) = \text{Var}(R_w) - \mathbb{E}[R_w]^{2+\alpha} \quad (6)$$

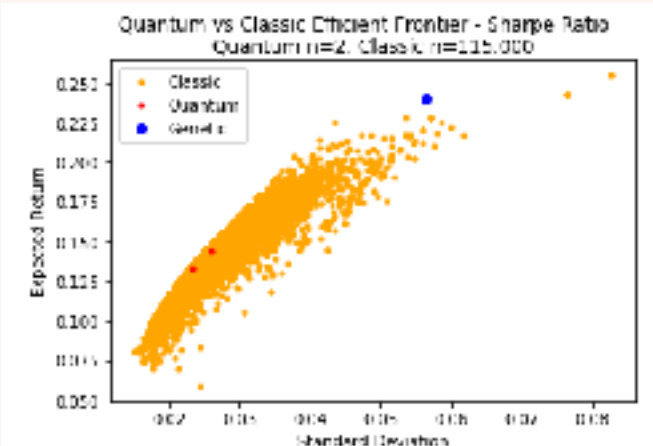
Where R_w is a weighted portfolio and $\alpha \in \mathbb{R}$ In most experiments we choose an equal weighting i.e. $w_i = 1/n$ where n is the number of assets included, and we choose α near 1. These are not requirements, but they do make the computations on DWave slightly easier. There is a wide open question as to finding optimal weighting and optimal α .

NEW FORMULATION (2)

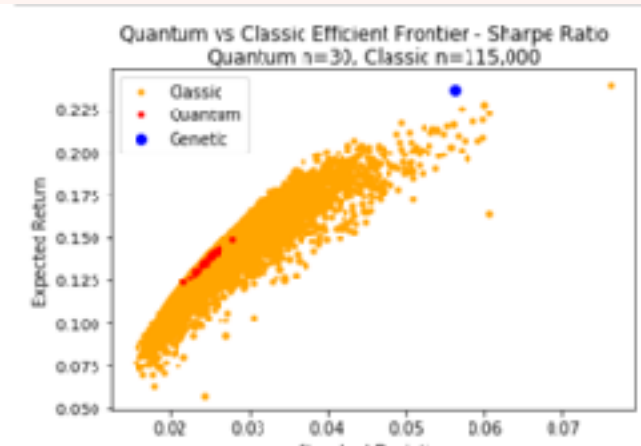
➤ The CQNS formulation on a quantum annealer finds portfolios near the Sharpe Efficient Frontier (via Monte Carlo)

2,588 Portfolios

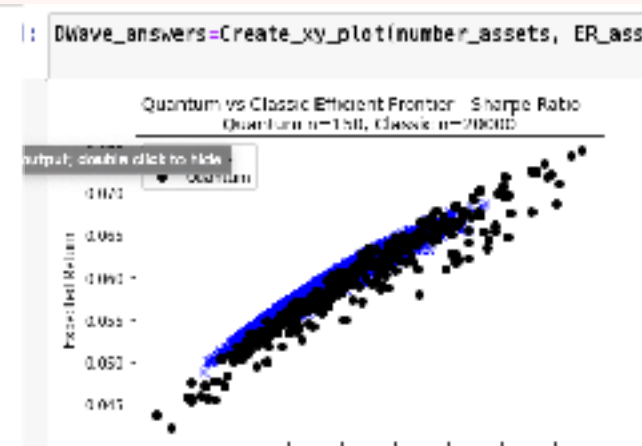
2 Portfolios



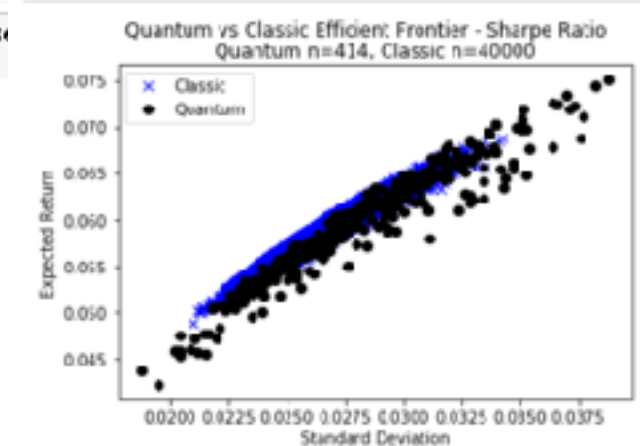
30 Portfolios



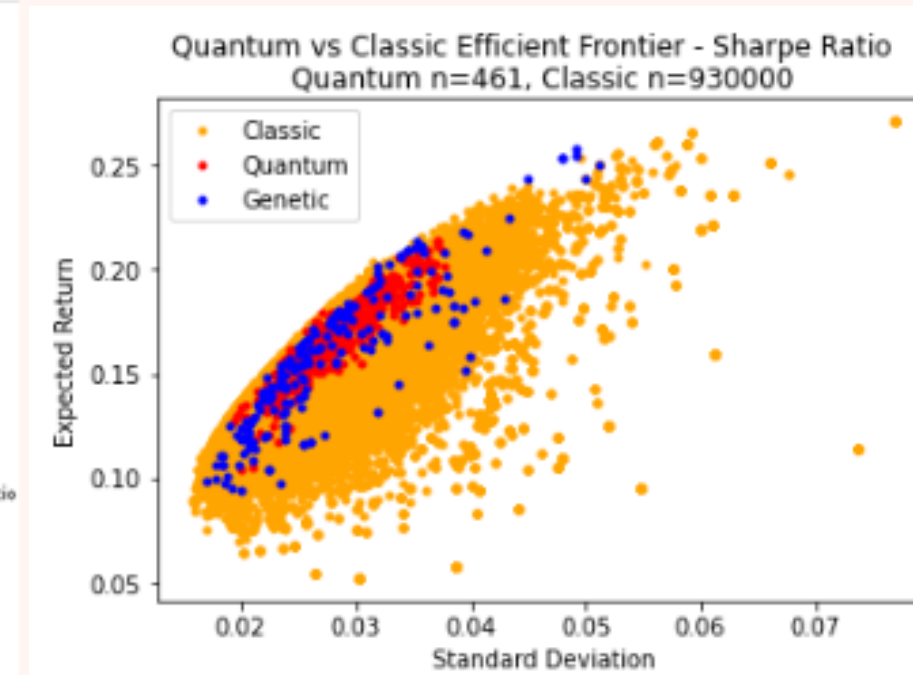
150 Portfolios



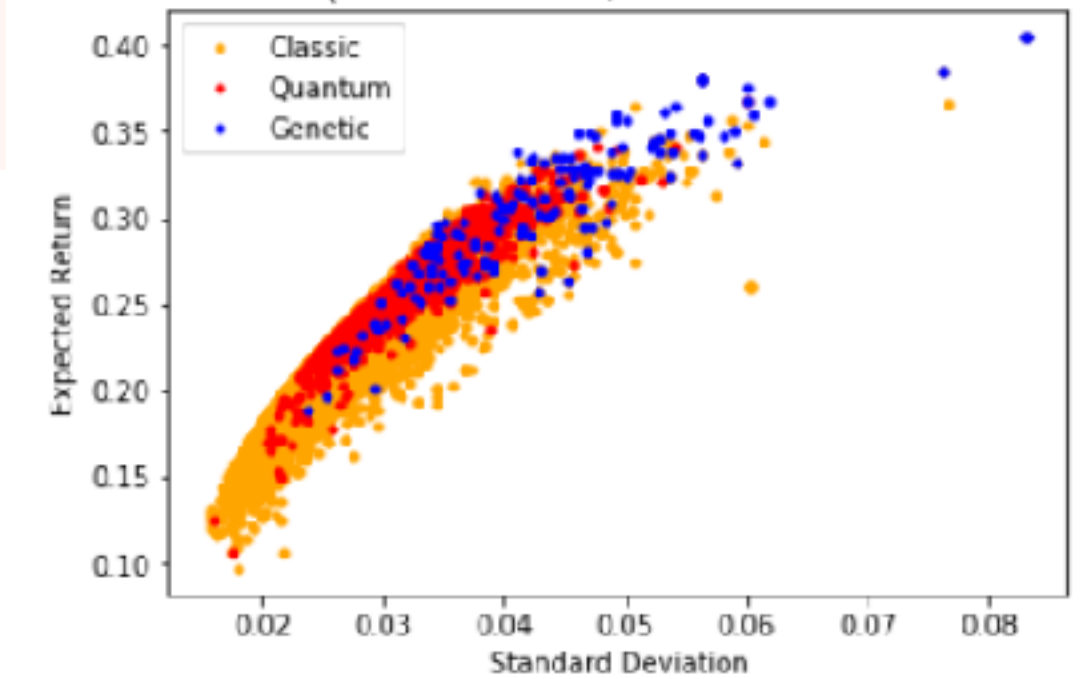
414 Portfolios



461 Portfolios (vs. 930k)

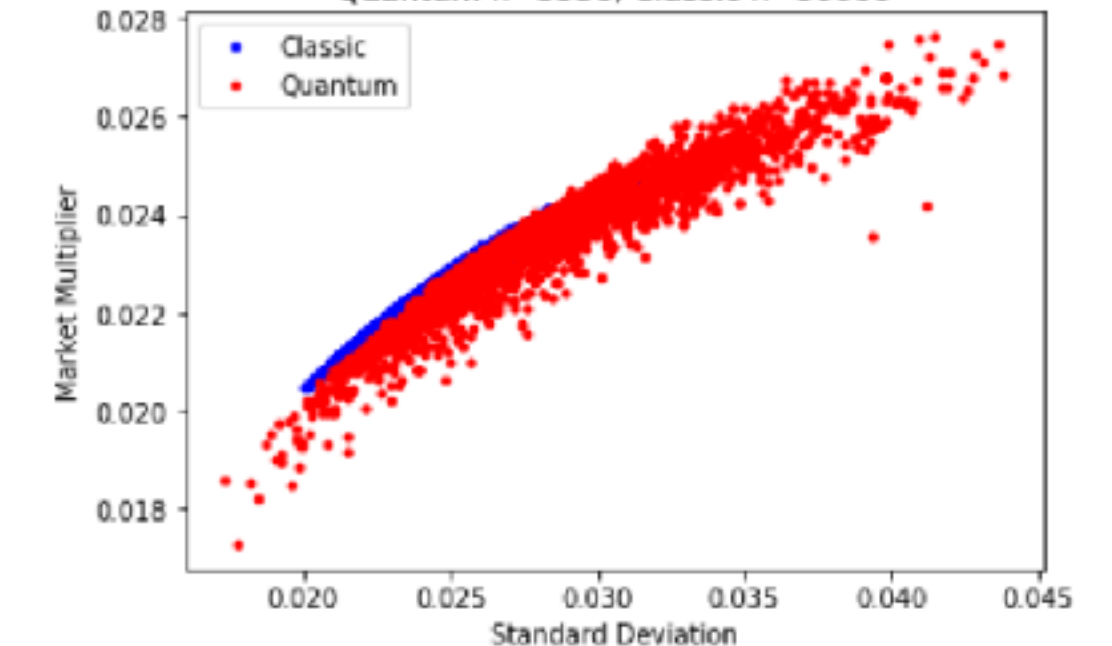


Quantum vs Classic Efficient Frontier - Sharpe Ratio
Quantum n=2588, Classic n=221660

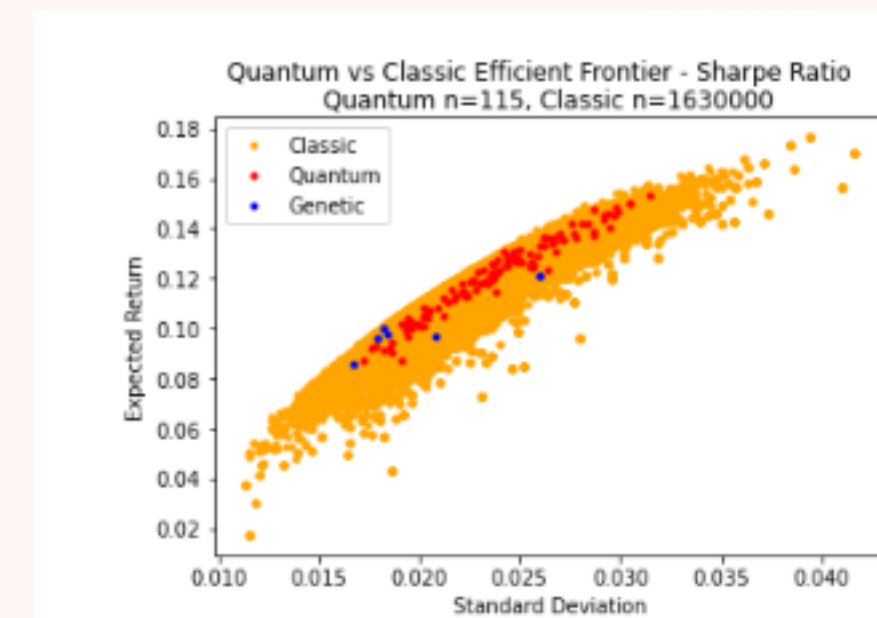


3,556 Portfolios

Quantum vs Classic Efficient Frontier - Chicago Quantum Ratio
Quantum n=3556, Classic n=50000



115 Portfolios (vs. 1.63M)



WHAT DID WE ACCOMPLISH?

- **Kept our focus on US liquid equities**
- **Solve 40-assets on a D-Wave 2000Q quantum annealing computer & classically**
- **Solve 60-assets on a D-Wave 2000Q & add classical capabilities**
- **Solve 64-assets on a D-Wave 2000Q**
- **Understand challenges to continued scaling**
 - **64 stock maximum on D-Wave 2000Q (2017) Chimera**
 - **Potential for more stocks on D-Wave Advantage (2020) Pegasus**
 - **Need faster methods (e.g., custom Simulated Bifurcation Model, QWOG) & better code**

INITIAL ANALYSIS

- Six methods 'solved' 60 stock problem in < 1 minute by finding "Ideal" portfolio
- These use non-proprietary methods (Python) & D-Wave Systems Inc.

Comparative Analysis of Methods

Method	Find Ideal CQNS	Seconds	Notes
Genetic Algorithm (Random)	Yes	7	456 seeds, 40 generations, 40 solutions / generation
D-Wave Simulated Annealer	Yes	11	Found best portfolio shown; modified energy levels / scores
Bespoke Simulated Annealer	Yes	15	Tuned down from 23 seconds; further tuning possible
D-Wave Quantum Annealer	Yes	21	Includes system parameter calibration time. 35% of 60 seconds in system run time. Expected to be 15 seconds moving forward.
Monte Carlo (Fat Tailed)	Yes	24	Ran 221,662 samples
Genetic Algorithm (D-Wave)	Yes	48	2588 D-Wave seeds, 40 generations, 40 solutions / generation
D-Wave Tabu Sampler	No	267	Scores found from 20-40 assets regardless of QUBO penalty
D-Wave Hybrid Sampler	No	5	No valid portfolios found, unsuccessful runs

Details on the Analysis

Category	Data	Notes
Date & Time of Market Data Download	August 5, 2020 15:00 EST	254 consecutive days of trading information
Ticker Symbols	AA AAL AAPL ABBV ABT ADBE ADI ADM ADP ADSK AES AFL AIG AJG ALGN ALK ALL ALXN AMAT AMD AMGN AMP AMZN APA ASML ATR ATVI AVGO AXP BA BAC BAX BBY BC BEN BHC BIIB BK BKR BLK BMRN BMY BP BRK-A BSX C CAG CAT CB CCI GDNS CERN CF CHKP CHRW CHTR CL CLF CLR PYPL	Same tickers as prior research paper
Risk Free Rate (%)	Calculated: 0.99% Used: 1.00%	
S&P 500	Calculated: 16.92% Used: 16.92%	
Russell 2000	Calculated: 2.88% Used: 2.88%	
Wilshire 5000	Calculated: 16.48% Used: 16.48%	
NASDAQ Composite	Calculated: 42.12% Used: 42.12%	
Overall Market Return	18.60%	
Variance of Market Benchmark	0.00045105	S&P 500
BETA Range (60 stocks)	{0.417, 2.12}	
All Asset Portfolio (60 stocks)	Expected Return: 22.09% Variance: 0.0006136 Standard Deviation: 2.48% Chicago Quantum Net Score: -0.013368 Chicago Quantum Ratio: 0.9129 Sharpe Ratio: 8.92	We are using a CQNS score from a formulation that is made to fit on a QUBO. A 'cleaner' formulation that runs classically has a value of -0.010161

"Ideal" portfolio minimizes:

- $CQNS = Variance - Return^{2+\alpha}$

Sharpe alternative maximizes:

- $CQR \approx Sharpe Ratio = Cov(i_m) / StDev$

INITIAL RESULTS

Comparative Analysis of Analysis Methods

Method	Find Ideal CQNS	Seconds	Best Portfolio Found	CQNS Score	Notes
Genetic Algorithm (Random)	Yes	7	APA	-0.05948	458 seeds, 40 generations, 40 solutions / generation Allowed 1 - 60 asset portfolios
D-Wave Simulated Annealer	Yes	11	AMP APA	-0.02899	Simulated annealer modifies energy levels / CQNS scores Allowed 2 - 59 asset portfolios
Bespoke Simulated Annealer	Yes	15	APA 2nd best: AMP APA CLR PYPL	-0.05948 -0.04069	Tuned down from 23 seconds; further tuning possible Allowed 1 - 60 asset portfolios
D-Wave Quantum Annealer	Yes	21	AMP APA	-0.05283	Includes system parameter calibration time. 35% of 60 seconds in system run time. Expected to be 15 seconds moving forward. Allowed 2 - 59 asset portfolios
Monte Carlo (Fat Tailed)	Yes	24	APA 2nd best: AMP APA	0.05948 -0.05283	Ran 221,862 samples Allowed 1 - 60 asset portfolios
Genetic Algorithm (D-Wave)	Yes	48	APA	-0.05948	2588 D-Wave seeds, 40 generations, 40 solutions / generation Allowed 1 - 60 asset portfolios
D-Wave Tabu Sampler	No	267	AA AAL AAPL ABT ADBE ADI AES AIG ALGN ALK ALL AMAT AMGN AMP APA ATVI AXP BAC BHC BIIB BKR BLK BRK-A BSK C CAT CDNS CERN CHKP CHTR CLF CLR	-0.00487	Scores found from 20-40 assets regardless of QUBO penalty Allowed 2 - 59 asset portfolios
D-Wave Hybrid Sampler	No	5	NA - No valid solutions found	NA	No valid portfolios found, unsuccessful runs
All 60 Assets (all-in)	NA	NA	stocks= 'AA AAL AAPL ABBV ABT ADBE ADI ADM ADP ADSK AES AFL AIG AJG ALGN ALK ALL ALXN AMAT AMD AMGN AMP AMZN APA ASML ATR ATVI AVGO AXP BA BAC BAX BBY BC BEN BHC BIIB BK BKR BLK BMRN BMY BP BRK-A BSX C CAG CAT CB CCI CDNS CERN CF CHKP CHRW CHTR CL CLF CLR PYPL'	-0.01337	Calculated classically in Python Only calculate 60 asset portfolio
Method	Find Ideal CQNS	Seconds	Best Portfolio Found	CQR Score	Notes
D-Wave Quantum Annealer	NA - CQR	21	CQR (best): AMZN BMY CQR (2nd): ALG AMGN CCI CHKP PYPL CQR (3rd): ABBV ABT ADBE ADM AMAT ASML ATVI CL BLK BP	1.03768 0.97904 0.97337	Same 2,588 D-Wave solutions. CQR calculated classically during post-processing.

- Our 'ideal' results: hold 2 of 60 stocks (APA & AMP)
- We selected 2 alternative portfolios based on 'good' CQR and CQNS scores
- Interpretation: pick high BETA stocks with offsetting covariance

INITIAL RESULTS WERE OPTIMISTIC

- **Market test after 5 weeks**
 - **CQR & CQNS 'adjusted' portfolios (6 or 7 assets) track benchmarks**
 - **CQNS "ideal" portfolio outperformed all benchmarks**
 - **Market advances pressure formulation to choose too few stocks (e.g., 2)**
- **We adjusted our formulation & picked 2nd portfolio**
 - **Set floors & ceilings on market indices**
 - **Adjusted CQNS power setting (reduce weight of expected return)**

Quantum Stock Portfolios



Chicago Quantum
Jul 15 - 8 min read ★



Portfolio	CAGR	# Assets
S&P 500 Index:	6.18%	1 ^GSPC
CQNS stocks:	5.32%	7 ADI AMP APA BA BAC BHC BLK
CQR stocks:	6.89%	6 AMZN ATR BAC BMY CERN CHRW
Ideal stocks:	13.93%	2 AMP APA
All stocks:	7.01%	60
AA AAL AAPL ABBV ABT ADBE ADI ADM ADP ADSK AES AFL AIG AJG ALGN ALK ALL ALXN AMAT AMD AMGN AMP AMZN APA ASML ATR ATVI AVGO AXP BA BAC BAX BBY BC BEN BHC BIIB BK BKR BLK BMRN BMY BP BRK-A BSX C CAG CAT CB CCI CDNS CERN CF CHKP CHRW CHTR CL CLF CLR PYPL		
Start and End Dates: 2020-07-10 through 2020-08-18		

New stock picks using the Chicago Quantum Net Score (classical methods)



Chicago Quantum
Aug 31 - 3 min read ★



```
Method Best Score Seconds Taken
All Assets -0.000397 x
['ADBE', 'IBM', 'ORCL', 'AXP', 'BA', 'BABA', 'BAC', 'BRK-B', 'BX', 'CCK', 'CHNG', 'CHTR',
'CNC', 'CLR', 'CNX', 'CPRT', 'DHR', 'DIS', 'DK', 'EL', 'ESTC', 'FB', 'FSLY', 'GE', 'GILD',
'GOLD', 'GOOG', 'GSK', 'HLT', 'INTC', 'JD', 'JNJ', 'LB', 'LNG', 'MSFT', 'MO', 'AZN', 'NIO',
'NVDA', 'NWL', 'PAYX', 'PFE', 'PK', 'QFIN', 'RACE', 'REGN', 'ROP', 'ROST', 'SBUX', 'SERV',
'SHW', 'SNE', 'MRO', 'SNY', 'STMP', 'TAK', 'TSLA', 'UAL', 'UBER', 'VALE', 'VIPS', 'W',
'WBT', 'TWTR']
MC Random -0.001711 94 ['AXP', 'CHTR', 'QFIN', 'W']
GA Random -0.002009 24 ['AXP', 'CHTR', 'QFIN', 'WBT']
GA D-Wave -0.002009 20 ['AXP', 'CHTR', 'QFIN', 'WBT']
SA Beskoke -0.001890 50 ['AXP', 'CHTR', 'QFIN', 'WBT', 'TWTR']
SA D-Wave 0.000248 65 ['BABA', 'GE', 'QFIN', 'ROP']
QA D-Wave -0.001497 time TBD ['AXP', 'CHTR', 'GE', 'LB', 'NVDA', 'WBT', 'TWTR']
```


BENCHMARK PERFORMANCE

- **First portfolio was run and published on July 9, 2020**
- **Initial returns over 2 and 5 weeks were at or better than the benchmarks**
- **CQNS returns @ 48 trading days were significantly below benchmark. Chicago Quantum Ratio outperformed benchmarks over timeframe.**

~25 trading days

Portfolio	CAGR	# Assets
S&P 500 Index:	6.18%	1 ^GSPC
CQNS stocks:	5.32%	7 ADI AMP APA BA BAC BHC BLK
CQR stocks:	6.89%	6 AMZN ATR BAC BMY CERN CHRW
Ideal stocks:	13.93%	2 AMP APA
All stocks:	7.01%	60
AA AAL AAPL ABBV ABT ADBE ADI ADM ADP ADSK AES AFL J TR ATVI AVGO AXP BA BAC BAX BBY BC BEN BHC BIIB BK BI CHKP CHRW CHTR CL CLF CLR PYPL		
Start and End Dates: 2020-07-10 through 2020-08-18		

48 trading days

Portfolio	CAGR	# Assets
S&P 500 Index:	6.787%	1 ^GSPC
CQNS stocks:	-2.318%	7 ADI AMP APA BA BAC BHC BLK
CQR stocks:	6.879%	6 AMZN ATR BAC BMY CERN CHRW
Ideal stocks:	-0.821%	2 AMP APA
All stocks:	4.439%	60
AA AAL AAPL ABBV ABT ADBE ADI ADM ADP ADSK AES AFL AIG AJG ALGN ALK AL L ALXN AMAT AMD AMGN AMP AMZN APA ASML ATR ATVI AVGO AXP BA BAC BAX BBY BC BEN BHC BIIB BK BKR BLK BMRN BMY BP BRK-A BSX C CAG CAT CB CCI CDNS CERN CF CHKP CHRW CHTR CL CLF CLR PYPL		
Start and End Dates: 2020-07-10 through 2020-09-16		

BENCHMARK PERFORMANCE (2)

- **Second portfolio was run and published on Friday, August 28, 2020**
- **Initial returns after 13 trading days, in a declining market show the CQNS portfolio of 4 stocks declined more than the market benchmarks (64 stocks & S&P 500)**
- **Decline of 3.357% vs. 2.798% (64 stocks) and 3.045% (^GSPC)**

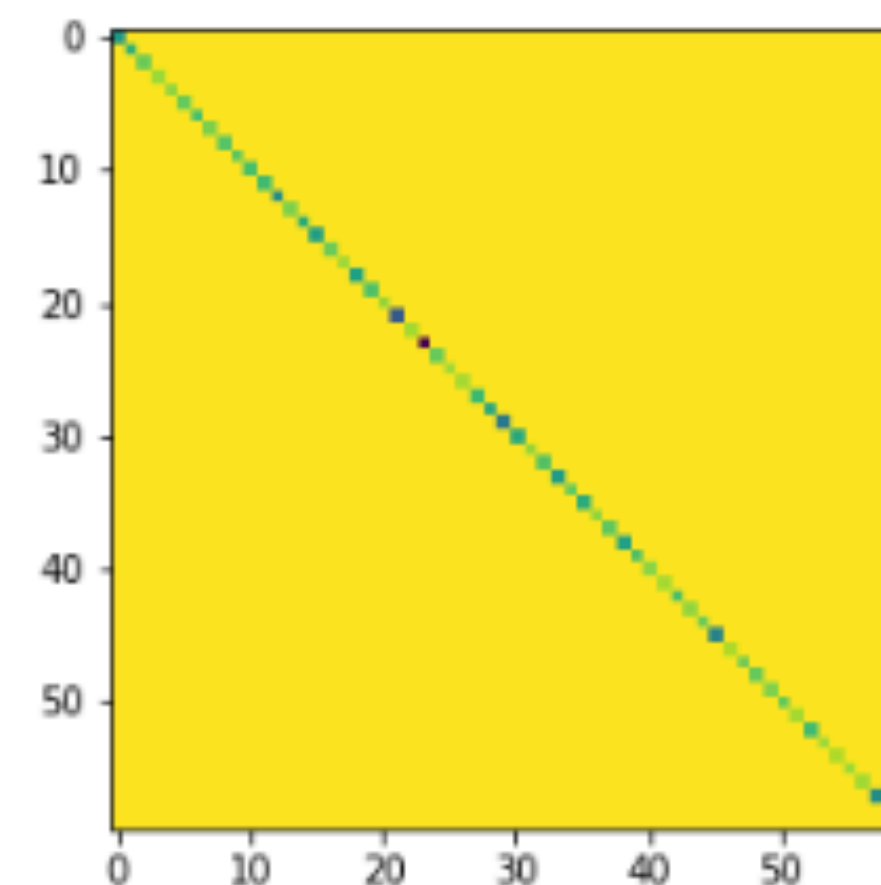
```
Portfolio          CAGR  # Assets
S&P 500 Index:    -3.045% 1 ^GSPC
CQNS stocks:      -3.357% 4 AXP CHTR QFIN WBT
All stocks:       -2.798% 64
ADBE AXP AZN BA BABA BAC BRK-B BX CCK CHNG CHTR CLR CNC CNX CPRT DHR D
IS DK EL ESTC FB FSLY GE GILD GOLD GOOG GSK HLT IBM INTC JD JNJ LB LNG
MO MRO MSFT NIO NVDA NWL ORCL PAYX PFE PK QFIN RACE REGN ROP ROST SBUX
SERV SHW SNE SNY STMP TAK TSLA TWTR UAL UBER VALE VIPS W WBT
Start and End Dates: 2020-08-28 through 2020-09-16
```

SOME INTERESTING DETAILS

SCALE CHALLENGE

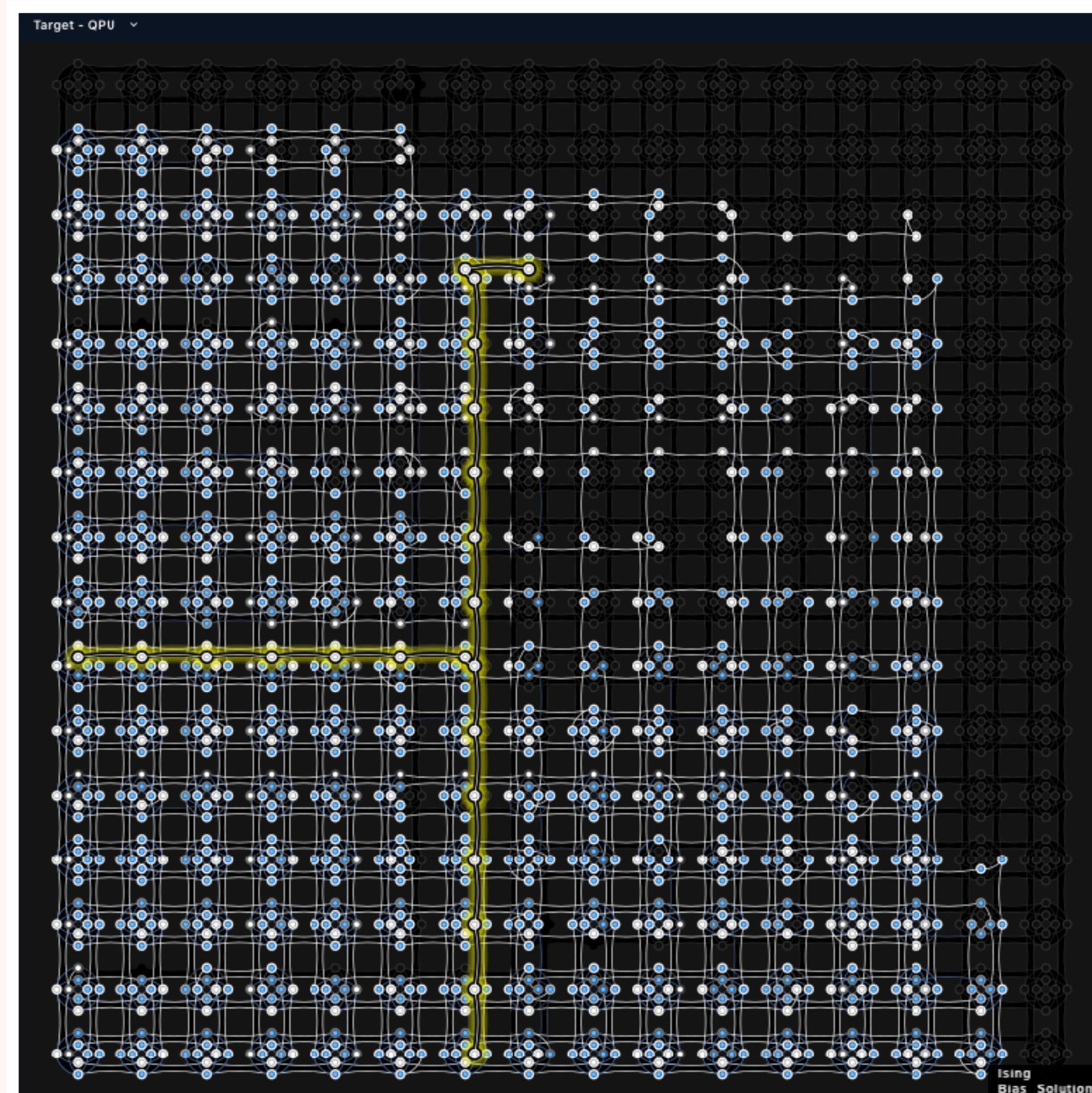
- **Our linear and quadratic terms need to be within $(-2, 2)$, and should be large enough to create an interesting energy landscape**

```
bigmatrix for 59
=====
[[-0.46793665  0.00424234  0.00372904 ...  0.00425073  0.0045156
  0.00372661]
 [ 0.00424234 -0.33661959  0.00365948 ...  0.00416498  0.00426864
  0.0036371 ]
 [ 0.00372904  0.00365948 -0.22624394 ...  0.0037191  0.00381722
  0.00368287]
 ...
 [ 0.00425073  0.00416498  0.0037191 ... -0.49012206  0.00438471
  0.00375156]
 [ 0.0045156  0.00426864  0.00381722 ...  0.00438471 -0.75830353
  0.0038178 ]
 [ 0.00372661  0.0036371  0.00368287 ...  0.00375156  0.0038178
 -0.25222184]]
=====
```



SCALE CHALLENGE

- **We use between 1300 and 1700 qubits out of 2048 installed for 60 assets. Yellow line represents one asset spread across 20 qubits (this is a 58 of 60 asset QUBO)**
- **Limited connectivity between 8 qubit cells requires chain lengths of up to ~40 qubits, which break**



SCALE CHALLENGE

- We can find up to a 64 clique embedding (fully connected vertices or stocks) that will run in the D-Wave *FixedEmbeddingComposite* solver
- There is no work-around to this (theoretical maximum is 65 stocks on the chimera graph)
- We expect Pegasus architecture can embed more than 64 stocks

```
# Find Clique Embedding using the D-wave, save it, then use it later
```

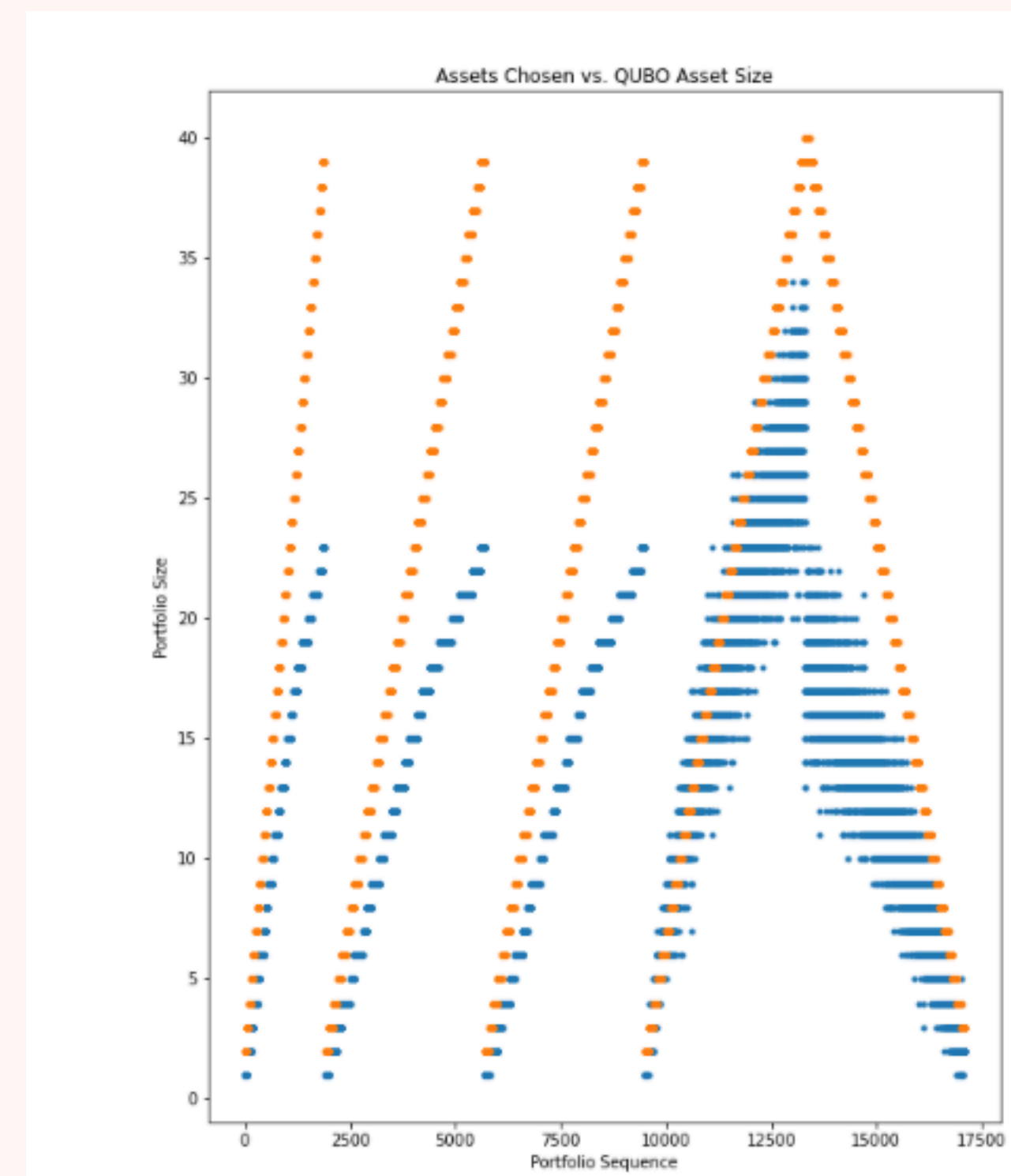
```
embedding_found = find_clique_embedding(64, 16, 16, 4)
print("Here is the embedding we found for our QUBO", "\n", embedding_found)
```

```
Here is the embedding we found for our QUBO
{0: [20, 16, 144, 272, 480, 528, 656, 784, 912, 1040, 1168, 1296, 1424, 1552, 1680, 1808,
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```

ENGINEERING ACCOMPLISHMENT

- **We can see the portfolio sizes we would like to see (orange dots) vs. blue dots (actuals). Dots should converge or be close (vertically).**
- **If not, we can adjust parameters and try again**
- **In this example, first three runs were unsuccessful, fourth was significantly improved, and 5th did 'ok' in reverse order**

D-Wave Systems Simulated Annealer Runs

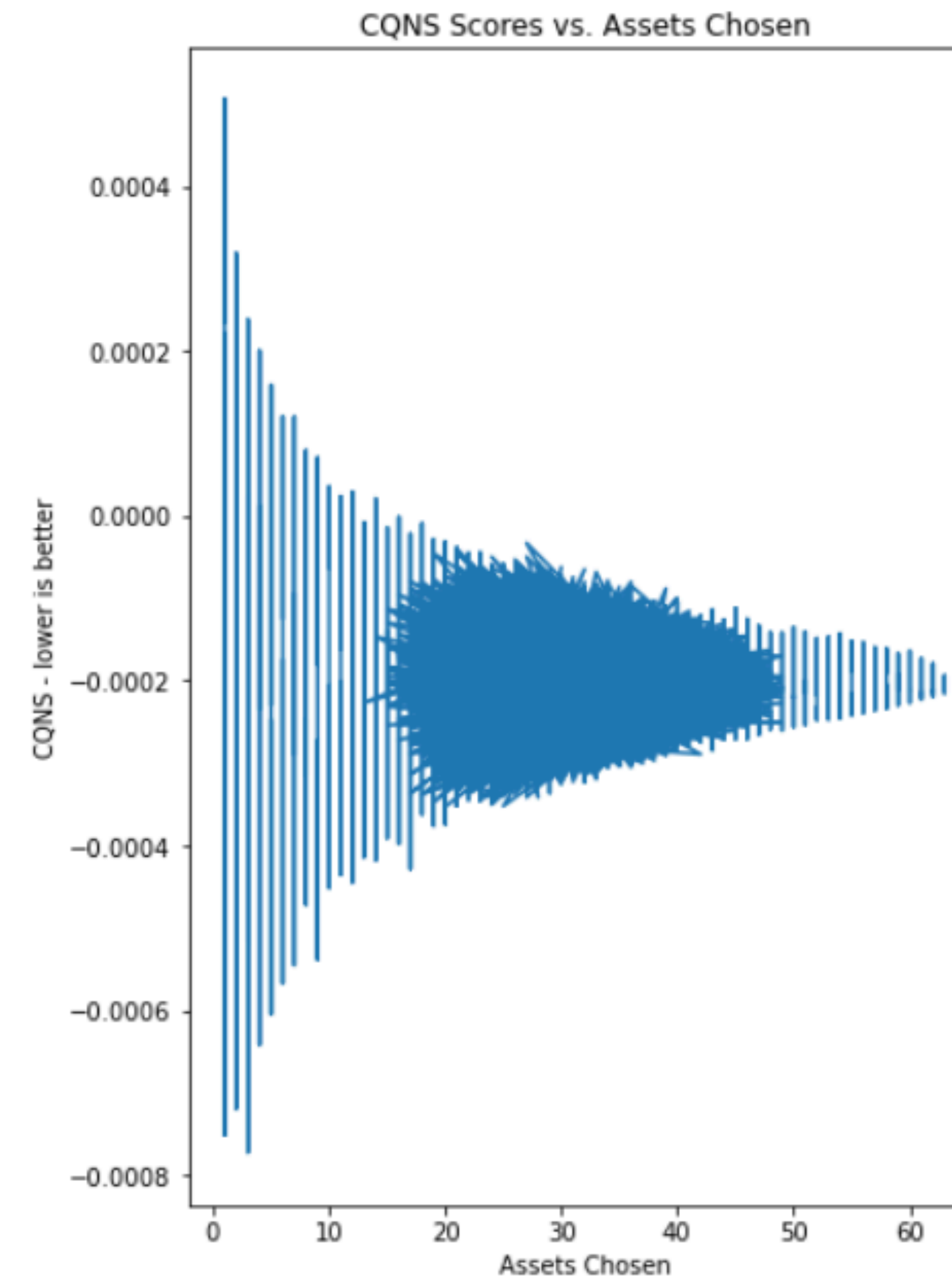


ENGINEERING ACCOMPLISHMENT

- **Our Monte Carlo Fat-Tailed approach combines a random sampling around $N/2$ assets with a small sample at each size portfolio (out of 64)**
- **This shape indicates where the optimal portfolios may reside. Different for each sample. For this sample, there are smaller portfolios with great CQNS scores**
- **We still search the entire space (2-64 assets)**

CQNS Scores as found by our fat-tailed Monte Carlo approach

Chicago Quantum Net Score (lower score is better)

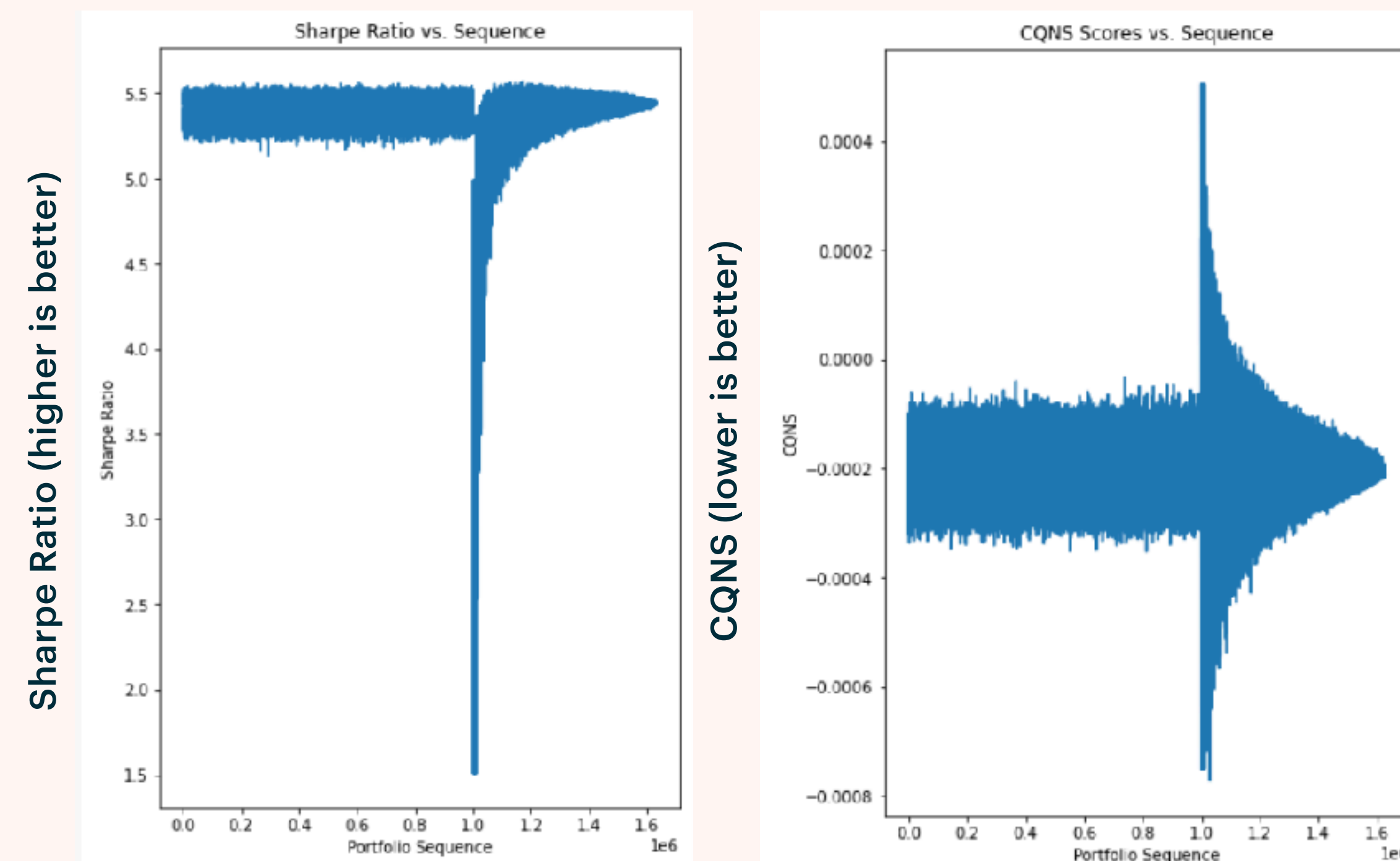


Portfolio sizes increase after initial random seeding

VISUALIZATION ACCOMPLISHMENT

- We can see the Sharpe Ratio 'best values' are hard to find (close to the top), and the best X% solutions are likely equivalent for an investor (left chart)
- The CQNS allows one to find 'great' and 'terrible' portfolios concentrated in the smaller portfolios (the vertical range) which quickly moves to a tighter range with random diversification (right chart)
- We create these views from our Monte Carlo Fat-Tailed approach (2,64) stocks

Sharpe Ratio & CQNS scores vs. random (N/2) & random by portfolio size



Portfolio sizes increase after initial random seeding (n=930k samples)

RESEARCH ARTIFACTS

- **Research Artifacts: 40 Stocks (arXiv), 4 x BLOG articles, including 60 Stocks Portfolio (Medium), 60 Stocks (arXiv)**
- **Research Gate**
- **Google Scholar**
- **Public Citations and Acknowledgements (from our website)**

WHAT ARE OUR 2020 GOALS?

- **Mature and expand our portfolio optimization model and application**
 - **Cap at 64 stocks on 2017 D-Wave quantum chimera architecture (2048 qubits)**
 - **Find maximum classical stock universe (1,855 takes ~ 1 hour)**
 - **Add simulated bifurcation model & quantum walks on graphs**
 - **Evaluate additional investment types & investment horizons**
 - **Add backtesting, independent portfolio tiling, and design UX/UI**
- **Earn revenues by serving clients to grow 'quantum' academic research capabilities**

MEET OUR CHICAGO QUANTUM TEAM

Twitter Name: @chicago_quantum

Website: <https://www.chicagoquantum.com>

US Advanced Computing Infrastructure Inc.

DUNS Number: 117153073

GAGE Number: 8D6D8

BBB Rating: A+

LinkedIn Profiles of the Chicago Quantum attendees:

<https://www.linkedin.com/in/jeffrey-cohen-2050053/>

<https://www.linkedin.com/in/alexkhanmba/>

<https://www.linkedin.com/in/clark-alexander-266232aa/>

Our team has one equity founder and multiple associates.

Complementary and advanced skillsets across academia, management consulting, financial services, sourcing, and IT.



Jeff Cohen

- President & Founder. Executive management consultant and corporate general manager.
- Built & led IT infrastructure consultancies (top: \$447M sales).
- Economics and Finance BA, MM and worked towards Ph.D. Masters in Management
- IBM, HPe, Siemens, McKinsey & Company and KPMG consulting leadership roles.



Clark Alexander, Ph.D.

- Mathematician, lead associate.
- Data analytics. Experienced big data analytics, forecasting, and research consultant.
- Helps clients structure their problems / forecasting needs into math and ETF/pre-process data to complete analysis.
- Mathematics PhD, former professor, data analytics consultant & quantum computing author.



Alex Khan

- Senior IT management professional with 20+ years and expertise in creating & maintaining applications for enterprises
- Certificate in Quantum Computing
- Executing IT and digital business transformations
- Product & program management
- Experienced in insurance IT
- Strong management consulting skillset...takes the executive CIO mindset
- Two Masters Degrees (MBA and MS Mechanical Engineering)

THE SYSTEM WE BUILT

APPLICATION PROCESS

- **We offer CQNS-based quantum & classical portfolio analysis for up to 64 stocks, for \$150. Classical only: \$50.**
- **Client provides 65-70 tickers which we validate (e.g., positive BETA, 253 trading days of data). We tune system parameters for specific portfolio, perform classical runs (MC, GA, SA, TABU), run quantum annealer (multiple embedding options), then run final genetic algorithm.**
- **Perform analysis and write-up results**
- **Currently takes a few hours to turn around the analysis**
- **Client signs up and pays on CQ website, emails tickers to CQ**

<https://www.chicagoquantum.com/portfolio.html>

APPLICATION OVERVIEW (1/3)

- 1. Download the data**
- 2. Understand the market over the past year**
- 3. Calculate the 'all in' 64 asset benchmark**
- 4. Run Monte Carlo**
- 5. Run GA...through D-Wave**
- 6. Run GA again (seeded by prior answers)**
- 7. Show the results...voting by method**

APPLICATION OVERVIEW (2/3)

1-3

4-6

```
[*****100%*****] 68 of 68 completed
Days and stocks, should be 252 or 253, number_assets (253, 64)
if less than 252 check for interrupted stocks & replace
[*****100%*****] 1 of 1 completed
shape of the rows & columns: 253 68
shape of the download (253, 68)
shape of the covariance data: (253, 64)
Number of assets: 64 or portfolios searched: 10 ^ 19
```

```
Actual riskfree rate = 0.78%
Use floor riskfree rate = 1.00%
Actual GSPC return = 12.64%
Use actual S&P500 rate = 12.64%
Actual RUT return = -1.64%
Use floor Russell 2000 rate = 2.50%
Actual W5000 return = 12.68%
Use actual Wilshire 5000 rate = 12.68%
Actual NASDAQ return = 34.99%
Use ceiling NASDAQ Composite rate = 20.00%
=====
Market return = 10.95%
```

```
ALL ASSET PORTFOLIO - equally weighted
Expected Return = 11.53%
Expected Return cubed - true 0.00052072
Expected Return cubed - qubo 0.00065997
Variance = 0.00044875
Standard Deviation = 2.12%
Chicago Quantum Net Score - true and qubo -7.1968e-05 -0.00021122
Chicago Quantum Ratio = 0.9863
Sharpe Ratio = 5.44
```

```
MC Discrete at N/2 Low Score Solution: -0.0003528497090690265 20 [[1 0
1 0 1 1 0 0 0 0 1 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 1 0 0 0 0 0
1 0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 1 1 1 0 0 0 0 0 0 1 0]] 1000000
=====
MC Low Score Solution: -0.0007721083847007409 3 [0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0] 1630000 Seconds
109
end of classical job
```

```
Best CQNS solution & portfolio size:
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0] -0.000772108384
7007409 Assets Chosen: 3
Best CQNS score found by GA vs. Monte Carlo, with delta: -0.000772 -0.0
00772 0.0
Did the GA find the same best solution? Yes if True ----->>>> True
Did the GA improve on Monte Carlo? Yes if True ----->>>> False
seconds to run: 47
```

```
Best from Genetic Algorithm - DWave Seed: -0.0007721083847007409
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0] Assets: 3
Best CQNS Negative score found by GA vs. MC, with delta: -0.000772 -0.0
00772 0.0
Did the GA find the same answer as MC? Yes if True ----->>>> True
Did the GA improve on MC? Yes if True ----->>>> False
seconds to run: 40
```


APPLICATION OVERVIEW (3/3)

7

```

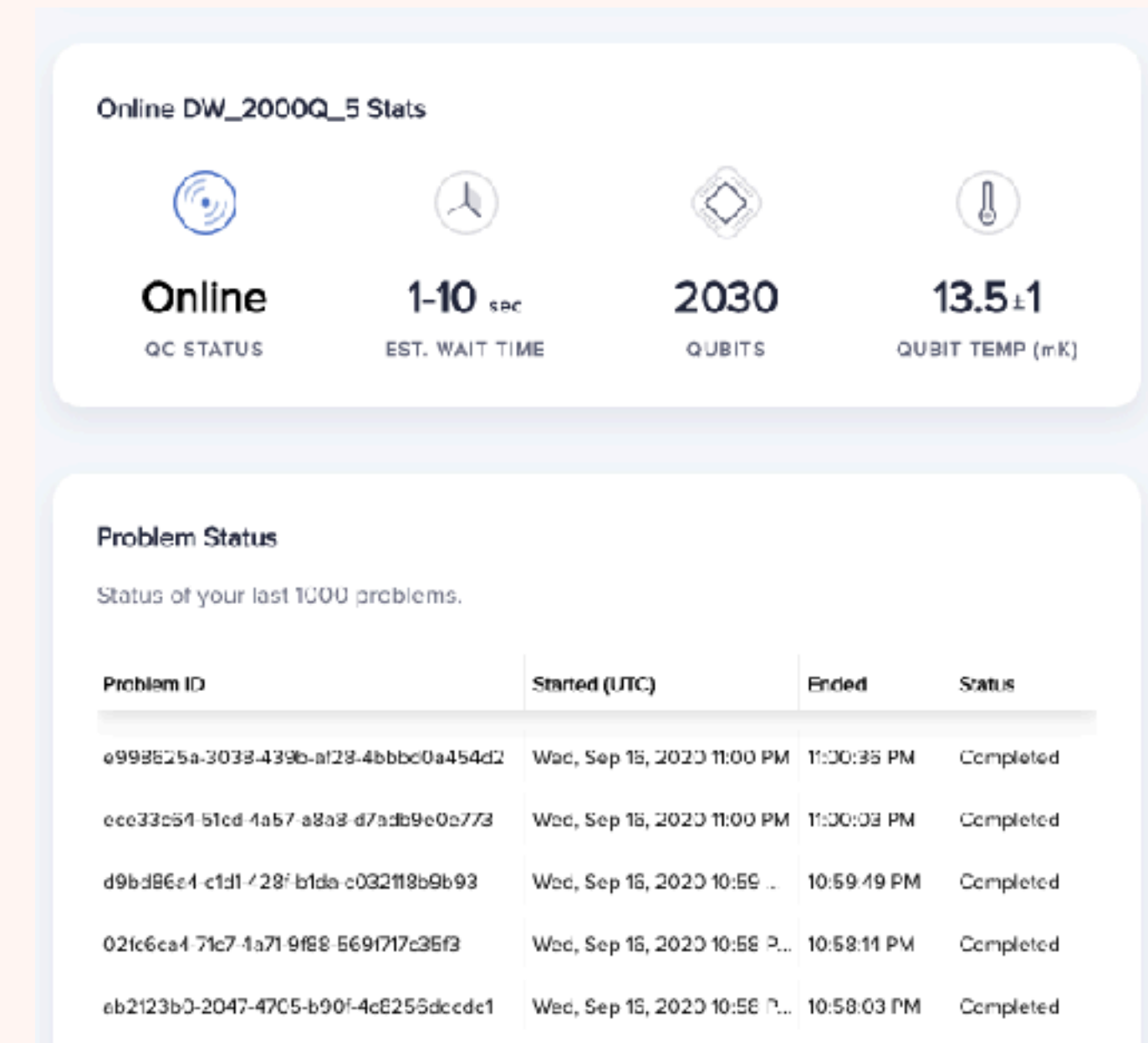
Method          Best Score      Seconds Taken  then show the stocks chos
en and the number of stocks
MC Random      -0.000772      000109
['XOM', 'LEG', 'TROW'] 3
GA Random      -0.000772      000047
['XOM', 'LEG', 'TROW'] 3
GA D-Wave      -0.000772      000040
['XOM', 'LEG', 'TROW'] 3
SA Beskoke     -0.000772      000111
['XOM', 'LEG', 'TROW'] 3
SA D-Wave      -0.000719      000085
['XOM', 'TROW'] 2
SA D-Wave      -0.000123      000083
['CTAS', 'XOM', 'PG', 'TROW'] 4
QA DW-all     -0.000581      TBD
['XOM', 'LEG', 'O', 'SYY', 'TROW'] 5
QA D-Wave      -0.000581      TBD
['XOM', 'LEG', 'O', 'SYY', 'TROW'] 5
TABU           -0.000286      000045
['MMM', 'AFL', 'APD', 'AMCR', 'T', 'ATO', 'CAH', 'CAT', 'CVX', 'CB', '
CTAS', 'CL', 'ED', 'EMR', 'BEN', 'GD', 'GPC', 'HRL', 'LEG', 'LIN', 'LOW
', 'PNR', 'PEP', 'PPG', 'RTX', 'O', 'SWK', 'SYY', 'TROW', 'TGT', 'VFC',
'WBA', 'GLD'] 33
All Assets     -0.000211      Zero
['MMM', 'AOS', 'ABT', 'ABBV', 'AFL', 'APD', 'AMCR', 'ADM', 'T', 'ATO',
'ADP', 'BDX', 'BF-B', 'CAH', 'CAT', 'CVX', 'CB', 'CINF', 'CTAS', 'CLX',
'KO', 'CL', 'ED', 'DOV', 'ECL', 'EMR', 'ESS', 'EXPD', 'XOM', 'BEN', 'GD
', 'GPC', 'HRL', 'ITW', 'JNJ', 'KMB', 'LEG', 'LIN', 'LOW', 'MKC', 'MCD'
, 'MDT', 'NUE', 'PNR', 'PBCT', 'PEP', 'PPG', 'PG', 'RTX', 'O', 'ROP', '
SPGI', 'SHW', 'SWK', 'SYY', 'TROW', 'TGT', 'VFC', 'GW', 'WBA', 'WMT',
'GLD', 'MSFT', 'AAPL'] 64

```

SOFTWARE OVERVIEW

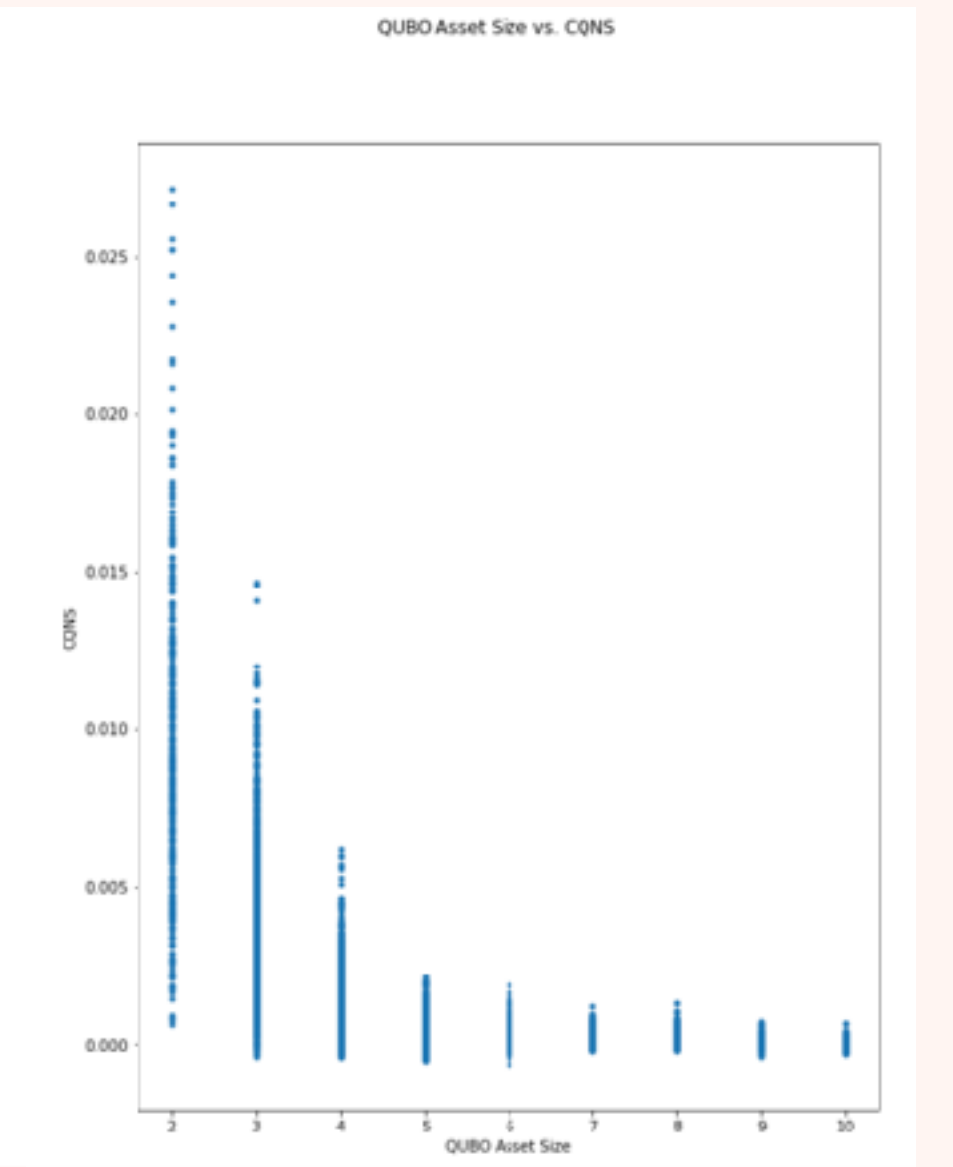
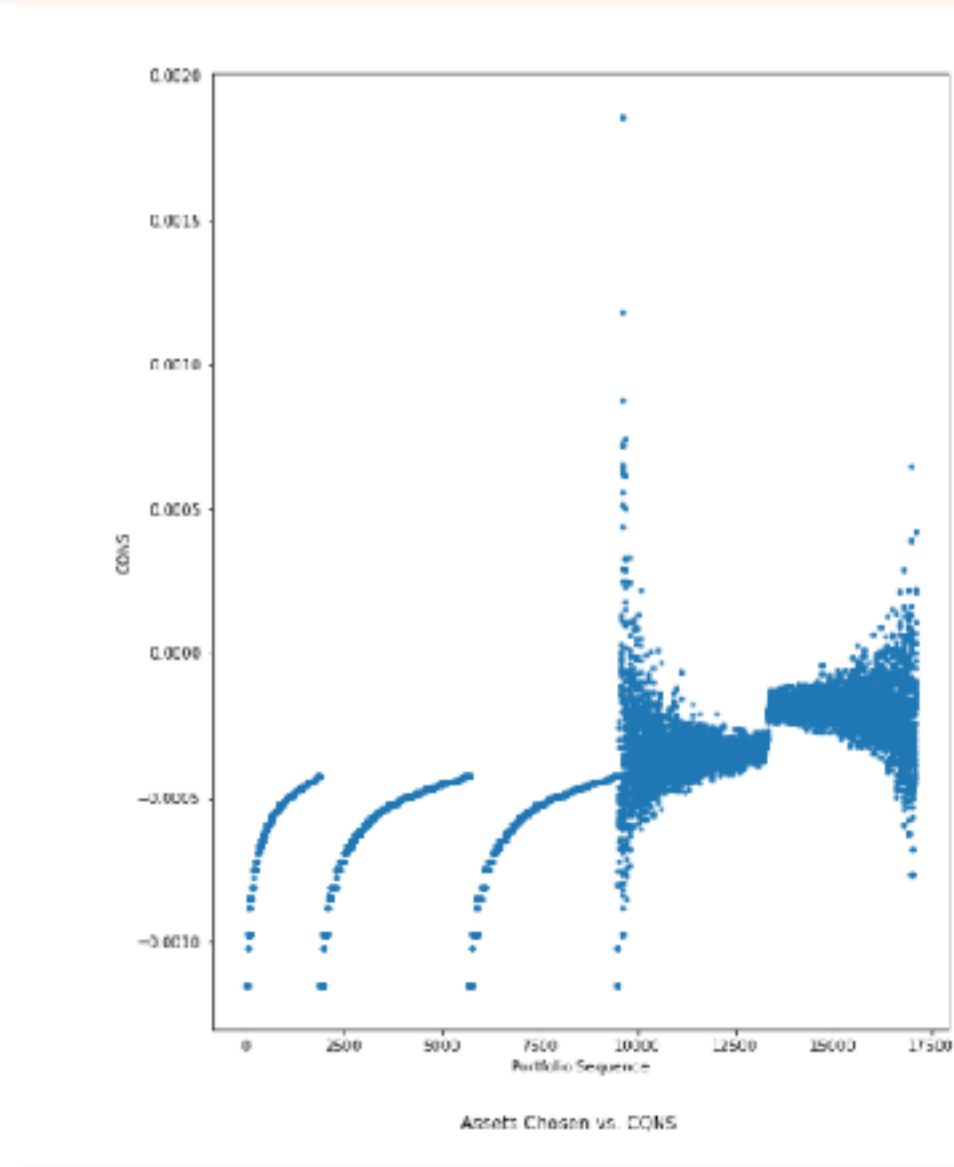
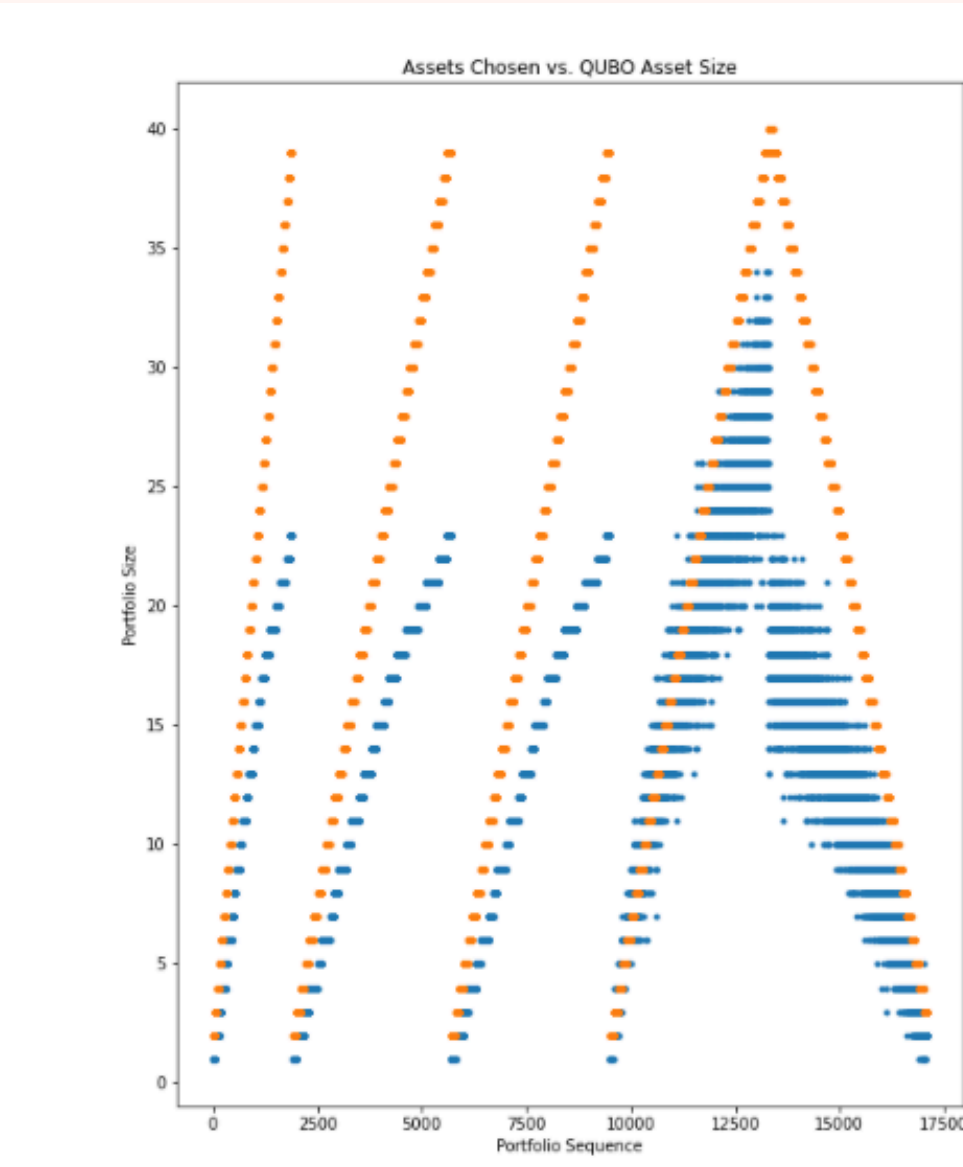
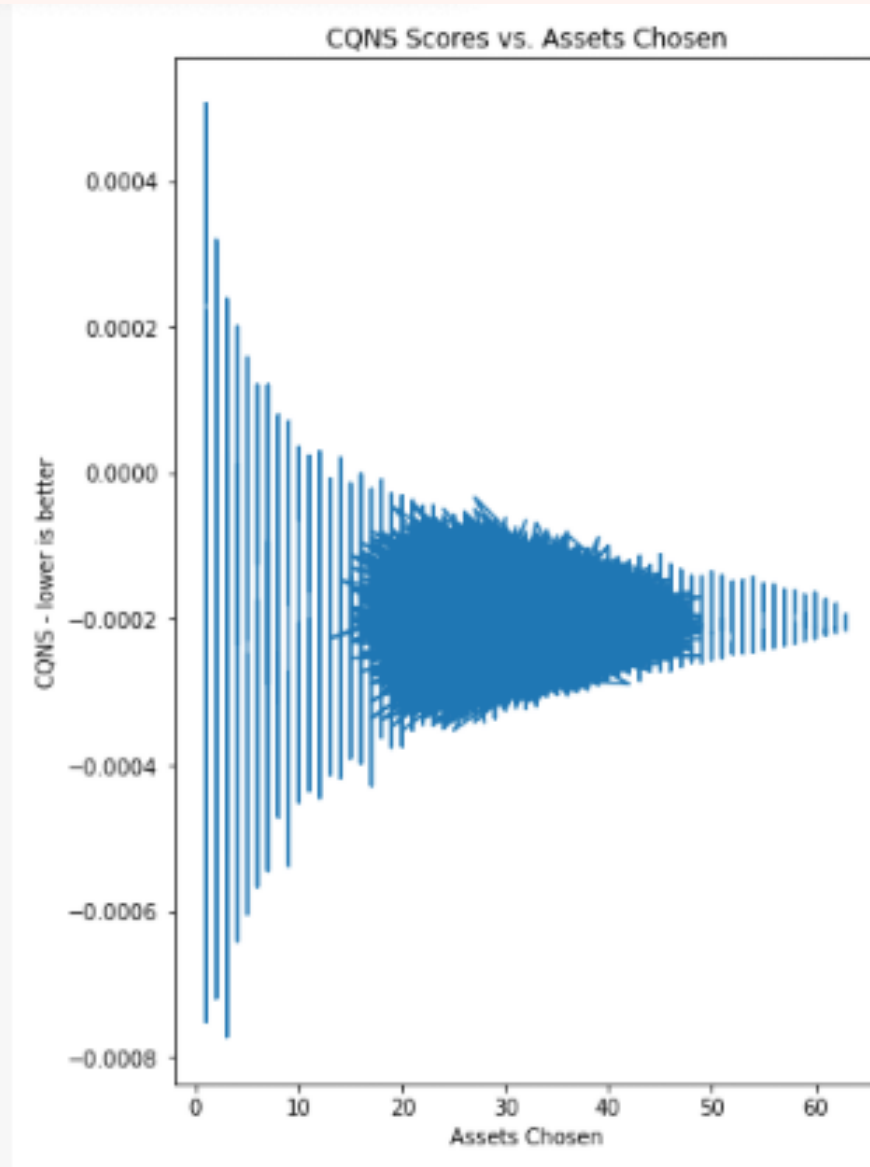
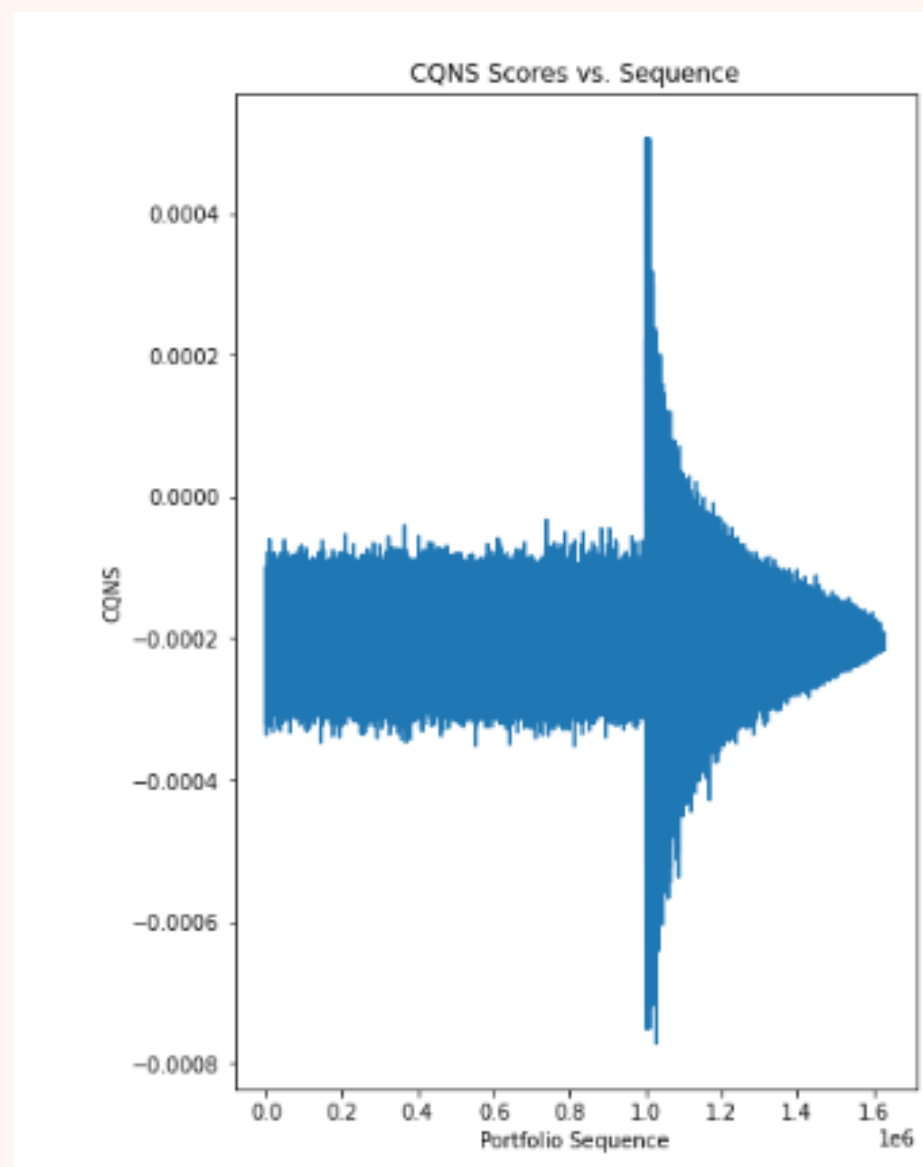
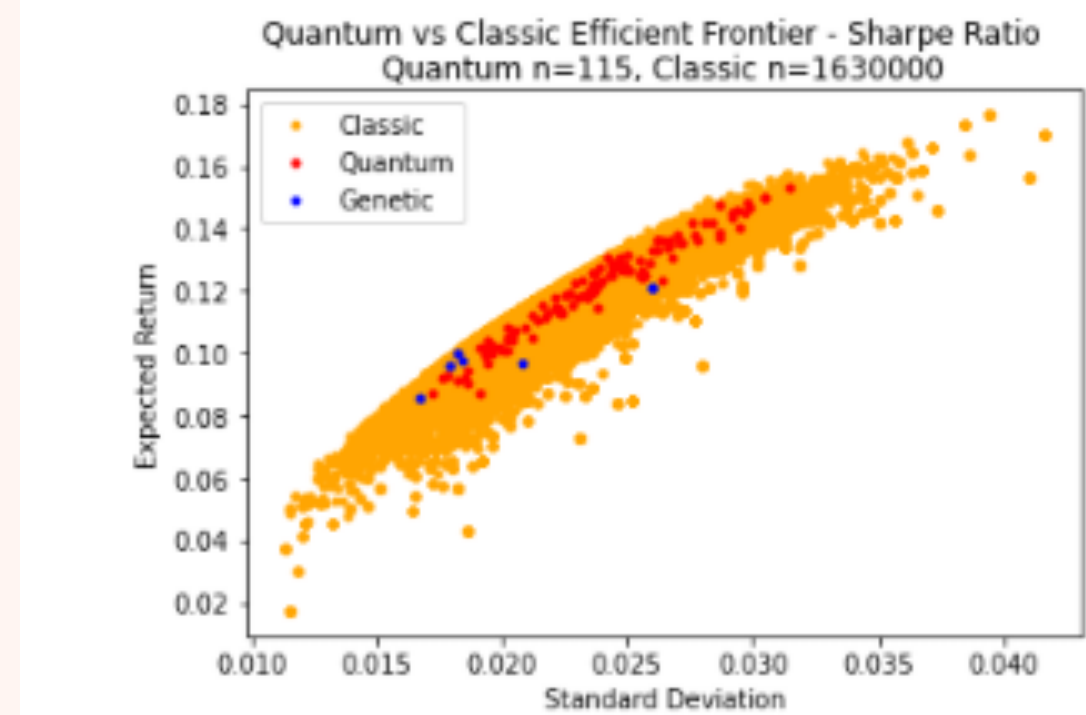
- **Coded in Python 3.7, runs on PC w/ backend to D-Wave Systems 2000Q**
- **FTP and validate tickers, tune parameters, build BQM/QUBO models, run custom & D-Wave classical models (MC, GA, SA, TABU)**
- **We call out to D-Wave (Canadian cloud) using the QUBO, return 'matching portfolio size' portfolios of (0,1) (equally weighted)**
- **Run D-Wave, soon all solutions through GA for deeper solution**
- **System has extensible plotting capability (matplotlib)**
- **Coming soon: finalizing custom simulated bifurcation model**

Backend System by D-Wave Systems 2000Q



SOFTWARE CHARTS & GRAPHS

Method	Best Score	Seconds Taken	then show the stocks chos
en and the number of stocks			
MC Random	-0.000772	000109	
['XOM', 'LEG', 'TROW'] 3			
GA Random	-0.000772	000047	
['XOM', 'LEG', 'TROW'] 3			
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['XOM', 'LEG', 'TROW'] 3			
SA Beskoke	-0.000772	000111	
['XOM', 'LEG', 'TROW'] 3			
SA D-wave	-0.000719	000085	
['XOM', 'TROW'] 2			
SA D-Wave	-0.000123	000083	
['CTAS', 'XOM', 'PG', 'TROW'] 4			
QA DW-all	-0.000581	TBD	
['XOM', 'LEG', 'O', 'SYY', 'TROW'] 5			



QUESTIONS?

Chicago Quantum

US ADVANCED COMPUTING
INFRASTRUCTURE, INC.

www.chicagoquantum.com