Time AND AGAIN

Many financial analysts make the mistake of thinking they live in a science fiction novel with numerous worlds, as **Professor Michael Mainelli, Chartered FCSI** explains

Nicolaus Bernoulli

THREE HUNDRED YEARS AGO, members of the legendary Bernoulli family elaborated the St Petersburg Paradox.

A casino proposes a game for a single player in which a fair coin is tossed repeatedly. The stake pot starts at £1 and is doubled every time a head appears. When a tail appears, the game ends and the player wins whatever is left in the pot. The player wins if the coin is tossed x times until the first tail appears – in other words, £1 if a tail appears on the first toss, £2 pounds if a head appears on the first toss and a tail on the second, £4 if a head appears on the first two tosses and a tail on the third, and so on.

The Bernoullis asked what would be a fair price to pay the casino for entering the game?

Assuming the casino has unlimited resources, the game can continue as long as the coin toss results in heads. Since a tail must come up sometimes, you are guaranteed to receive some payout (at least £1). Further, you can plot all the various permutations of heads and tails and see that, on average, you win, and win big (the expected gain is infinite). Yet people quickly grasp that this game offers large gains only at very low probabilities and typically offer small sums – say £25 – to play. What seems paradoxical is the discrepancy between what people are willing to pay to enter the game and the infinite expected value.

> ***The St Petersburg Paradox is no paradox, rather the wrong use of ensemble averages***

In a paper for the Royal Society 298 years later, Professor Ole Peters of the Santa Fe Institute and the London Mathematical Laboratory offered a solution from a viewpoint that affects the way we analyse financial situations today. He pointed out an obvious flaw in analysing the St Petersburg Paradox, using the common practice of gauging investments by the sum of all possible outcomes weighted by their probabilities. This is the expected return approach of financial analysts. Peters notes that expected return is equivalent to imagining multiple copies of ourselves in parallel universes, one for each possible future, and then averaging the results. Physicists call the average of all these multiple copies an ensemble average.

Time for time averages

To analyse the St Petersburg Paradox we should use another average – the time average – in which the future unfolds in just

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one universe over a long time. In order to do this we have to assume playing equivalent games repeatedly – that is, we judge the gamble as part of a long-term investment strategy, compounding returns over time. To make a simple numerical comparison – invest £100 subject to a 50% rise or fall with equal probability. Out of many parallel investments, half go up to £150, half down to £50, and on average nothing happens. The ensemble average is £100. But in a single investment over time the representative 50% rise to £150, followed by a representative 50% drop results in £75 after two rounds, a time-average loss of about 13% per round.

Imagine that you have paid £1m for a ticket, hoping for at least 20 consecutive heads. If tails shows up on the first toss, can you afford to continue? You live one life and can realise only one of the possible futures. If you lose, you can't ask your imaginary winning selves to make your losses whole and, if you win, you have no way to recompense your imaginary losing selves.

A few people do win a lot in the casino, but most play a very short game. As Peters notes: "The expectation value of my wealth in some gamble may be exponentially increasing while I'm guaranteed to go bankrupt." The St Petersburg paradox is no paradox, rather an inappropriate use of ensemble averages for analysis instead of time averages.

When considering multiplicative growth, the time perspective converges with the ensemble perspective when fluctuations are small. But with moderate or large fluctuations, the ensemble perspective will be different from the time perspective. Looking across markets such as equities or fixed income or commodities, there will be trends up or down. For the individual investor, the time perspective is relevant, and the ensemble view is not. A common strategy that addresses this problem is diversification and it may work for some, but for others only delays eventual failure.

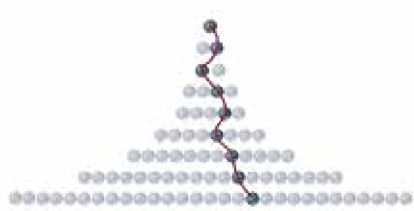
Above average analysis

Time-average analysis is markedly different from ensemble-average analysis. Economic models – for instance, most that exhibit economic growth as well as 'random walks' or geometric Brownian motion – should be using time averages. Towers Watson produced a paper looking at the practical implications of time averages. It concluded that Peters' new viewpoint affected the assessment of stochastic models, leverage limits (Sharpe ratios), risk-based asset allocation, catastrophe bonds and increased the importance of cashflow, speculating whether target returns might be affected too: "Just as the tortoise beat the hare in Aesop's fable, we have a hunch that a low-volatility, robust portfolio built on time averages will compound through time at a faster rate than a 'racy' portfolio built on ensemble averages."

Further, risk management and regulation using ensemble averages are called into question. Peters' colleague, Alex Adamou, explains: "Risk has two sources. The first is that the future is uncertain. The second is that time is irreversible. Choices about uncertain futures are risky because we can't go back and try again. We must live with the consequences. Without uncertainty or irreversibility, decisions would be trivial, and economics duller, if more accurate." Peters relates fluctuation to risk: "Ensemble averages do this funny thing: they get rid of fluctuations before the fluctuations really have any effect. That makes it very difficult to deal with risk because risk is often just another word for fluctuations." Peters and his colleagues have gone on to apply their approach to understanding leverage-driven bubbles and crashes, setting optimum leverage and explaining income inequality. Aiming high, they are well on their way to developing a decision theory as a branch of mathematical physics.

Peters asserts that the time average, in which the future unfolds in just one universe, is what we, as individuals, actually experience and is therefore a more appropriate measure on which to base our decisions. For financial analysts, we need to be clear whether we're analysing our client's time or the time of the multiverse. There is a big difference. ■

Possible worlds



Over time, the single path through possible worlds is more reliable than expected returns

With thanks to Ole Peters and Alex Adamou of the London Mathematical Laboratory.

For a list of relevant articles written by Ole Peters, please visit cisi.org/cpdparadox

A 30-minute discussion of these issues between Michael Mainelli and Ole Peters is available now on CISI TV. The CISI is planning a special 300th birthday event for the Paradox, to assess its importance for modern-day finance, on 9 September. For details, please visit **cisi.org/events**

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Professor Michael Mainelli, Chartered FCSI is Executive Chairman of Z/Yen Group. He co-founded Z/Yen, the City of London's leading commercial think-tank and venture firm, in 1994 to promote societal advance through better finance and technology. Michael's third book, co-authored with Ian Harris, *The Price of Fish: A New Approach to Wicked Economics and Better Decisions*, won the Finance, Investment & Economics Gold Prize at the 2012 Independent Publisher Book Awards